DESIGN OPTIMISATION BY INCREMENTAL MODIFICATION OF MODEL TOPOLOGY

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ABSTRACT

Using a standard commercial package, case studies in shape optimisation were used to investigate rules and heuristics for identifying design variables for shape optimisation and incremental updates to the topology of a geometric model from finite element stress results. Topological modifications based on the classical Euler operations were used. The results indicate that intelligent interpretation of analysis results could be used to identify design variables for shape optimisation and model changes for topology optimisation. Typically only a few incremental changes to the topology are required before the additional benefit of a given increase in complexity becomes small.

Keywords: topology optimisation, shape optimisation

1. INTRODUCTION

Techniques for topology optimisation, the creation of a design model with a new or modified topology, have mostly been based on homogenisation procedures [1, 2]. The main disadvantage of this approach is that a valid CAD geometric model has to be extracted from the element mesh generated by the homogenisation, which is rather like a gray-scale image. Furthermore, the technique is not well-suited to the incremental improvement of an existing design. Shape and topology optimisation have been seen as separate and distinct procedures.

The aim of this paper is to argue that candidate modifications to an existing topology can be derived by intelligent interpretation of the results from an initial analysis on a given geometry. With an incremental improvement strategy, the manufacturing costs of a given modification to the shape can be used to assess whether the increase in complexity is justified. Most analysis and optimisation is carried out on designs where the basic layout is well established and is known to be close to a practical optimum. Therefore, a procedure which could identify shape variables for design optimisation and suggest incremental improvements in topology is likely to be just as useful as a procedure which aims to identify a globally optimum topology within an arbitrary design space.

At this point, no attempt has been made to develop any software or automatic procedure for shape or topology improvement. The work which will be reported here is a series of case studies investigating candidate changes to the design shape and topology, illustrating how analysis results can be used to choose the most suitable modifications.

The Pro/Mechanica [3] package was used for shape optimisation of manually generated topologies. In all cases the objective was minimum weight, subject to an upper limit on the von Mises stress in the model in a single load case. This package provides p-adaptive analysis of automeshed geometry, so that convergence is guaranteed. An introduction to shape optimisation can be found at [4].

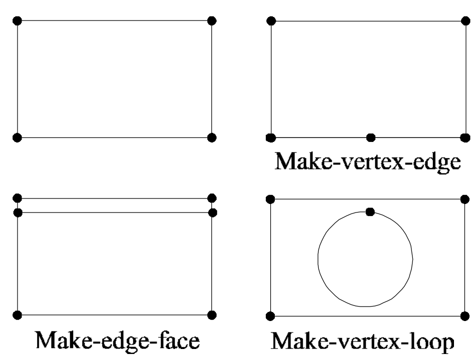

Figure 1. 2D Euler Operations

Topological modifications to design geometry can be enumerated in terms of the well-known Euler operations

[5, 6]. In terms of increasing the complexity of a 2D topology, this implies operations such as make-edge-vertex, make-edge-face and make-vertex-loop, Figure 1.

An alternative view is that these operations expand the definition of the boundary in a parametric solid model [7], creating new design variants.

For a 2D surface with *n* edges in a single outer loop, this implies that *n* new topological models can be generated by make-edge-vertex, that ${}^{n}C_2 = n(n-1)/2$ models can be generated with make-edge-face, and that 1 new model can be created with make-vertex-loop.

2. CASE STUDIES

2.1 Cantilever Beam

The first case study was a simple cantilever beam, fixed at one end and subject to a point load at the other. The first optimisation was to optimise the thickness of a simple rectangular plane stress model of the region, Figure 2(a).

Given a minimum thickness model, the stresses along the outer fibre of a rectangular section beam vary linearly with bending moment, being maximum at the support and zero at the free end. Since material is only fully stressed at the fixed end, a shape change is obviously necessary. Assuming that the beam depth at the free end is adjusted, a tapered cantilever is produced, Figure 2(b), in which the von Mises stress along an outer fibre varies as shown in Figure 3. Note that now the stress is large for a substantial fraction of the length. However the stress still decreases dramatically in the outer third of the beam. Since no further shape changes are possible with this topology, an increase in topological complexity is required to provide more design variables.

2.1.1 Edge subdivision for cantilever

The simplest topological change is to split one of the model edges using an Euler operation, make-edge-vertex, giving a beam with two tapers, Figure 2(c). The location of the new vertex and the beam depth at that point were added as design variables. The initial guess for the axial location was taken as the point where stress reached a maximum, since the shape optimisation needs additional freedom in this area to reduce the stress. The final position of the vertex is close to the initial position, implying that the initial position was close to a local optimum.

Figure 4 shows the variation in the outer fibre stress along the length of a beam with two tapers. Again, the new vertex was introduced at the location of maximum stress along the edge. Introducing a further vertex at the second stress maximum and optimising the result gives the model of Figure 2(d) and the outer fibre stress plot of Figure 5. The results of Table 1 show that the reduction in weight due to the extra complexity introduced by the second and third make-edge-vertex operations is minimal.

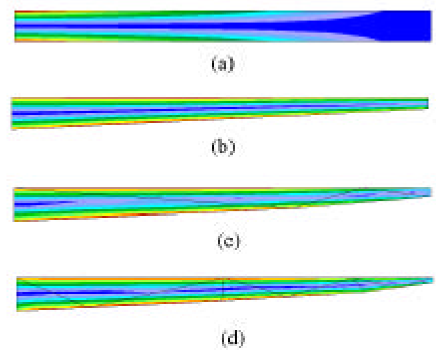

Figure 2. Von Mises stress in cantilever beam

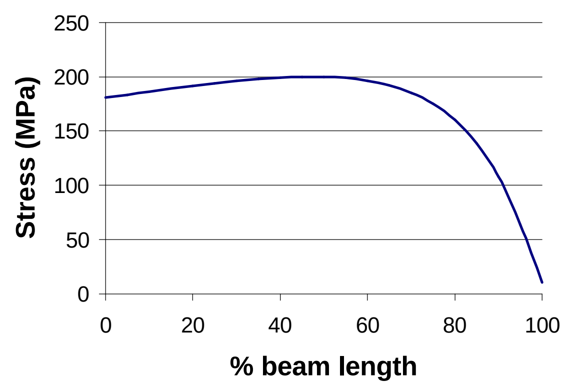

Figure 3. Outer fibre stress in tapered cantilever

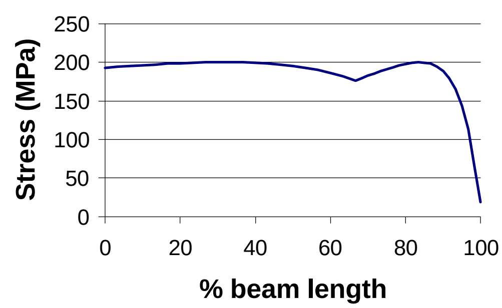

Figure 4. Outer fibre stress in cantilever with 2 tapers

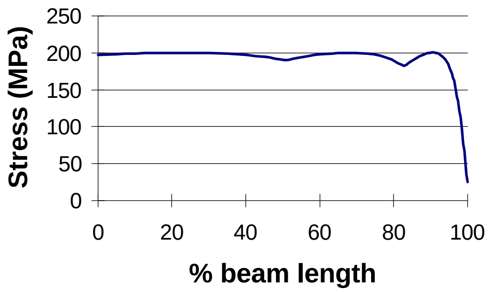

Figure 5. Outer fibre stress in cantilever with 3 tapers

2.1.2 Face subdivision for cantilever

An alternative type of topological change is to partition the original rectangular region into areas of different thickness using the make-edge-face operation of Figure 1. One application of this operation will result in a T-section beam, two operations would provide an I-section. (Alternatively, if the partitioning edges were vertical a cantilever with two different widths would be produced.) Figure 6 shows optimised sections with these forms. A constraint on the minimum plate thickness was enforced to simulate a local buckling constraint. Note that the reductions in weight achieved are much larger than were achieved with edge subdivision, Table 1.

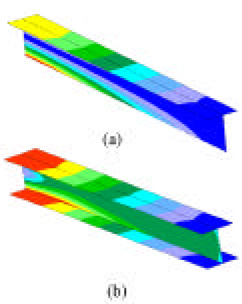

Figure 6. Beams Created by Face Subdivision

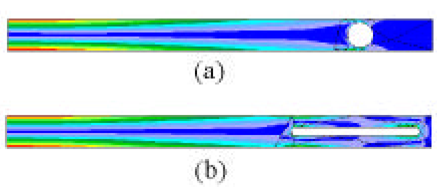

Figure 7. Hole and Slot Insertion

2.1.3 Loop insertion for cantilever

The other useful 2D topological change is to insert an interior loop of material inside a face. If the thickness of material in this new loop is zero, a hole results. Figure 7(a) shows the effect of inserting a hole close to the end of the beam and introducing the axial position and radius of the hole as design variables. Once this optimum has been achieved, two further divisions of the hole edge at the position of maximum von Mises stress allow a slot to be developed, Figure 7(b). The reductions in weight are again summarised in Table 1.

Design Modification	**Weight (%)**
Tapered Beam	71.1
Edge Subdivision	
2 straight lines	70.4
3 straight lines	70.0
Face Subdivision	
T section	70.4
I section	27.0
Loop / Hole Insertion	
Hole	96.0
Slot	90.8

Table 1 Optimisation Results for Cantilever Beam

2.2 Quarter Plate with Hole

The second case study was the standard stress concentration problem of a square plate of finite size with a central hole, subject to uniaxial tension. In this structure the stresses are two dimensional and relatively complex compared to the cantilever. Using a symmetric quarter model loaded in to horizontal direction, the design was first shape optimised by varying the radius of the hole to minimise weight subject to the same maximum von Mises stress constraint, Figure 8(a). Once this optimum has been found and given that the outer boundary of the plate is fixed, no further shape improvement is possible and a topological change is required to introduce more design freedom.

2.2.1 Edge subdivision for quarter plate

Figure 9 shows the variation in von Mises stress along the length of the circular arc defining the hole boundary. The stress minimum near the middle of the curve suggested that a design change would be beneficial near here. Introducing a new vertex to split the arc into two, a new shape optimisation was performed with both arc centre positions and radii as design variables. The resulting shape, shown in Figure 8(b), reduced the weight to 83% of the original plate. Note that the arcs defining the hole have not been constrained to run perpendicular to the symmetry planes.

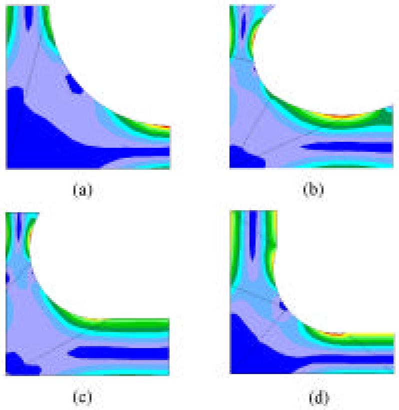

Figure 8. Quarter plate optimisations - edge subdivision

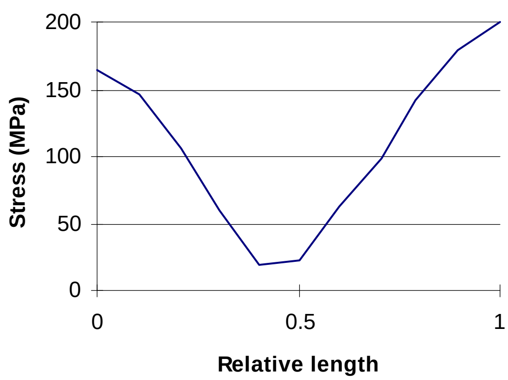

Figure 9. von Mises stress round hole perimeter

There are several interesting points to note. Firstly, the new point was positioned about half way along the length of the curve producing the hole. This was exactly where the minimum stress occurred in the original model. Secondly, the tangents of the two arcs are almost exactly identical at their meeting point. This result was found by the optimiser, and no constraint to force this was placed on the model. This is a desirable outcome which will prevent unwanted protruding corners and stress concentrations.

Further optimisation runs were performed with hole shapes defined by an arc and a straight line, Figure 8(c), and an arc with two straight lines, Figure 8(d). The reductions in weight are similar to those obtained with the two arc model, which is perhaps not surprising since the increase in topological and geometric complexity is similar. A table summarising the effect of all the shape and topology changes for this model is given in Table 2.

2.2.2 Face subdivision for quarter plate

Figure 8(a) shows there is an area in the bottom left of the plate which has low stress, suggesting that the plate thickness in this area could be reduced without a large adverse effect on the stresses. A model was created and optimised, and is shown in Figure 10(a). The design variables were: the radius of the arc dividing the two sections, the points at which the arc joined each outside edge of the plate and the radius of the hole. The hole radius decreased as did the thickness of the new section. This reduced the weight of the plate to 48.8% of the original weight. This is a dramatic reduction, more than halving the weight. It should be noted that the radius of the arc subdividing the face was extremely large which effectively made it a straight line.

Other subdivisions of the face are possible, but these were not studied exhaustively since a strategy for evaluating the most promising ones *a priori* is not clear at this point.

2.2.3 Loop insertion for quarter plate

Another approach to eliminating the lowly stressed areas in the bottom left corner of the model is to add a hole in this area. Figure 10(b), (c) and (d) show the effect of adding a hole to the original quarter plate model, the model with a hole described by 2 arcs and the model with a hole formed by 1 arc and 2 straight lines. The corresponding weight reductions are given in Table 2.

The most dramatic reduction in weight is achieved with the subdivision of the face into regions of separate thickness. These results are therefore similar to those observed for the cantilever beam case.

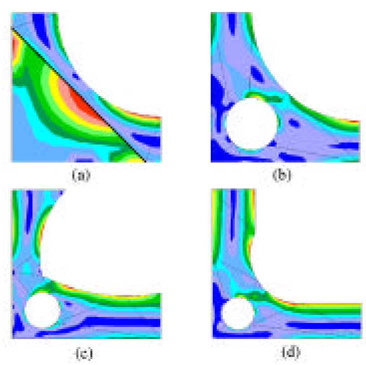

Figure 10. Quarter plate optimisations - face subdivision and loop insertion

Design Modification	**Weight (%)**
Edge Subdivision	
2 arcs	82.8
1 arc, 1 straight line	83.7
1 arc, 2 straight lines	83.3

Face Subdivision	
2 sections of different thickness	48.8
Loop / Hole Insertion	
1 arc + hole	84.4
2 arcs + hole	73.3
1 arc, 2 straight lines + hole	73.3

Table 2 Optimisation Results for Plate with Hole

2.3 Short Bracket

The third shape considered was a short angle bracket. A symmetric half of the initial design is shown in Figure 11(a). The plate was loaded with a vertical load distributed along the outer edge. An initial optimisation of the plate thickness showed that the largest stress occurred at the symmetry plane and that there was a large variation in stress along the length of the top edge. Given that a fully stressed design will have the same stress everywhere, this implies that a shape optimisation should aim to increase the stress at the outer vertex and decrease it at the inner end. A shape optimisation in which the position of the inner vertex on the symmetry plane and the x-y position of the outer vertex were varied gave the result of Figure 11(b).

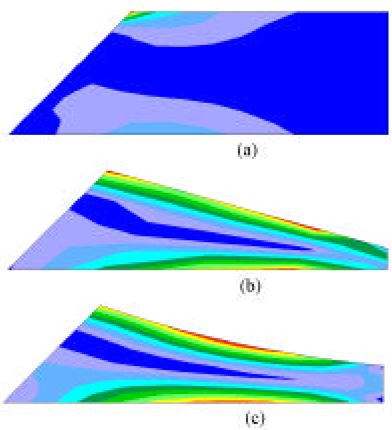

Figure 11. Short bracket shape optimisations

A further shape change which was considered in this case was to convert the straight line defining the taper into an arc. However the extra design variable did not prove beneficial and a bracket of exactly the same weight was produced, Figure 11(c). This is because the radius of the arc was very large in comparison to the bracket length.

A line subdivision at the point of maximum stress in the tapered bracket of Figure 11(c) was then applied to give a topology change. Initially, the straight line was constrained to be horizontal and this produced a bracket where the arc and line were exactly tangential. However, the weight of the model increased slightly. The model is shown in Figure 12(a). The horizontal constraint on the straight line was removed and a new optimisation reduced the weight to 54.2% of the original value, but this is only 2.1% lighter than the bracket with the single straight line, so the point of diminishing returns has obviously been reached, at least in a local sense.

The model shape is shown in Figure 12(b), and exhibits some interesting points. Firstly, the arc and straight line are almost tangential. Secondly, the arc meets the axis of symmetry at 98^{o}, so a nearly symmetrical model was found.

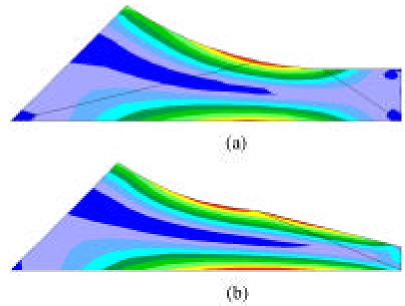

Figure 12. Line subdivision in short bracket

Other design changes would be to introduce holes in areas of low stress and regions of different thickness, but the results would almost certainly have been similar to those obtained with the cantilever and the quarter plate, so these alternatives were not investigated again.

3. DISCUSSION

The case studies considered here are very limited: the weight of some simple 2D shapes was minimised subject to a single stress constraint with no deflection constraints for a single load case. All the shapes were bounded by straight lines and circular arcs. As was stated in the Introduction, the objective was to investigate whether or not an incremental approach to adding topological complexity to a model might be a useful alternative or addition to the homogenisation methods which are gaining in popularity.

Specifying the appropriate shape optimisation variables for Mechanica was quite time consuming, even for the very simple models considered here. Models with more than a few design variables either took a long time to solve or else no feasible solution was found. Any heuristics which can identify design variables which are likely to have a significant effect would be useful. Some simple heuristics which can be suggested are:

1. In 2D plane stress optimisation, identifying areas of different thickness had the greatest effect

2. Stress maxima or minima at the end of an edge imply that the vertex position should be a design variable for

shape optimisation, assuming that the plate thickness has been adjusted so that the mean stress is acceptable

3. If the stress gradient is zero in the interior of an edge, adding a vertex at that point introduces additional topological complexity and design freedom at an appropriate place
4. Holes are most useful when the stress minimum is not on the boundary

Both bosses and holes may be regarded as regions bounded by inner loops of edges within a region, having a thickness which may go to zero. Bosses should be introduced in areas of localised high stress; holes in areas of low stress.

The geometry of the models studied was deliberately limited to straight lines and circular arcs to avoid the potential complexity associated with spline curves. However the definition of an arc via a centre point, start and end angles meant that it was not possible for the optimiser to cause the arc to degenerate to a true straight line or for the centre of curvature to move smoothly to the other side of the line. Alternative parametric forms of the circular arc [8] with these properties are known; these would be much more suitable for optimisation.

The results show that the law of diminishing returns clearly applies - once a few topological modifications have been made to the base structure the improvement in structural efficiency gained from further model complexity is negligible. With an incremental approach, the cost (in terms of manufacturing cost, assembly cost etc.) of any additional geometric complexity can be compared with the benefit gained from introducing it. Presumably there are equivalent heuristics for combining topological elements to reduce complexity - again the benefits in terms of manufacturing cost can be compared with any weight increase caused by the simplification.

The results presented do not attempt to define an algorithm for identifying features in a stress distribution and the corresponding optimum topological modifications. However, it is suggested that the calculation of design sensitivity to transformation of elements of the model boundary is a potentially productive and insightful approach to the problem.

Of course this incremental approach may never find a global minimum, but global optimisation is an extremely difficult problem and typically involves finding local minima in the vicinity of a distribution of starting points spattered throughout the feasible design space. Any measures which can help find geometric models at local optima will be helpful. The primary suggestion is that the finite element results should be associated with the higher level organisation of the geometric model to help chose an update to the geometric model shape and topology. A well defined algorithm has not yet been defined, but some simple guidelines can be distinguished.

The design changes which made the largest difference to the component weight were the introduction of regions of different thickness. Subdividing a face with n edges between 2 edges, the number of possible combinations is n(n-2)/2, so an exhaustive evaluation of all the possibilities would be expensive, though it might be possible to assess the sensitivity of the design to a given reinforcement economically using 1D edge stiffeners.

The extension of these ideas to 3D has not been attempted, but the benefits of being able to identify candidate shape optimisation variables and to modify a solid model topology incrementally are corresponding greater, especially compared to the difficulty of constructing a valid CAD model from a collection of finite elements.

4. CONCLUSIONS

The variation of finite element stress results over the faces and edges of a CAD model can be used to identify design variables for shape optimisation.

Incremental modifications to the topology of a geometric model can provide new design variables for shape optimisation. It appears that a rational strategy for identifying useful updates is feasible.

The cost and benefit of an incremental change to the topological complexity of a geometric model can be assessed much more easily than trying to identify the geometric model which corresponds to an optimised but unstructured finite element mesh.

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