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# New Techniques for Grid Generation and Adaptation

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**Abstract**

Several new techniques based on the deformation method are reported. These include techniques for domains with moving boundaries, boundary fitted triangulation with prescribed boundary nodes, adaptation of multi-block meshes and curvilinear triangular meshes.

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*Keywords:* Mesh Deformation; Multi-block; Curved Triangles;

## 1. Grid Generation and Adaptation

Over the past decades, numerical grid generation has been widely used in domain discretization for numerical solution of differential equations. Despite considerable success, grid generation and adaptation remain a challenging task for numerical simulation and manufacturing design with complex geometries. Indeed, a NASA sponsored study [6] states that *Mesh generation and adaptivity continue to be significant bottlenecks in the CFD workflow, and very little government investment has been targeted in these areas. As more capable HPC hardware enables higher resolution simulations, fast, reliable mesh generation and adaptivity will become more problematic. Additionally, adaptive mesh techniques offer great potential, but have not seen widespread use due to issues related to software complexity, inadequate error estimation capabilities, and complex geometries.*

The aim of our efforts is to research and develop innovative game-changing techniques for grid generation and adaptation that are suitable for aerodynamic simulation and design. We will focus on three types of grids (meshes) suitable for CFD on complex geometries:

1. **High order**, solution-based and geometry-based adaptive **unstructured** meshes.
2. Solution-based and geometry-based adaptive **multi-block structured** grids.
3. Cartesian grid-based, **body-fitted unstructured** grids (as demonstrated in Example 6 below).

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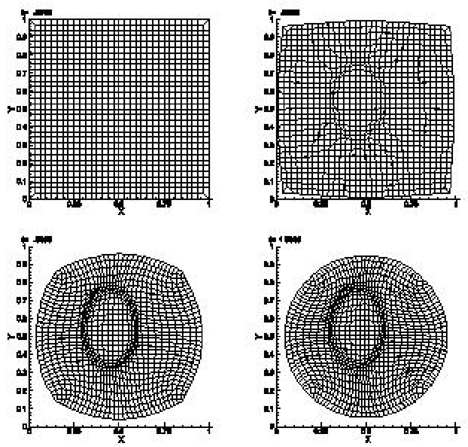


Fig. 1. Example 1

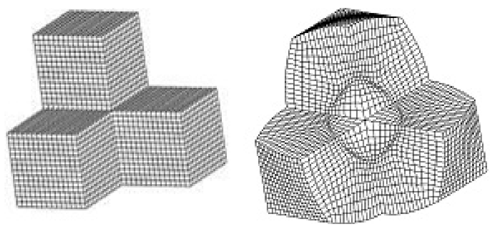


Fig. 2. Example 2

## 2. Proposed Techniques and Examples

Currently, high order unstructured meshes are generated by the elasticity model, which cannot guarantee positive Jacobian determinant and the effective ranges are very limited. Our proposed techniques overcome these shortcomings because they are based on an innovative deformation method. The deformation method has very large effective range and it can guarantee prescribed Jacobian determinant. Therefore, it is simple, effective and reliable. In fact, it is a stand-alone grid generator representing a significant advance in grid generation methodology. This is because we generate a grid on domains of curved boundary by deforming an initial grid on a simple domain. Some mathematical details are provided in [1],[3],[4] below.

### *2.1. The deformation method*

The deformation method has its origin from differential geometry. It generates a transformation $T$ with prescribed Jacobian determinant specified by a monitor function $f(x, y, z, t)$. Thus $J(T) = f(T, t)$. All examples presented in this note, except Examples 1 and 5 are new and unpublished.

**Example 1:** Generating an adaptive structured grid on a disk by deforming a grid on a square with a prescribed size distribution. See figure 1.

**Example 2:** Deforming a multi-block grid on four cubes to an adaptive grid on a curved body. See figure 2.

**Example 3:** A linear grid of only 9 cells on a quarter disk (Figure 3d) is generated by deforming a Cartesian grid on a square (Figure 3a) as the left and bottom boundary nodes are moved to the arc in ten time steps.

**Example 4:** Generation of linear and curved triangular grid on a quarter disk (Geometry-Based Adaptivity). In this example, the initial linear grid on the square is subdivided into a triangular grid (see Figure 4a). This initial triangular grid is deformed in ten time steps into the linear triangular grid on the quarter disc D that is colored in red (see Figure 4b). Midpoints on sides of triangles in the initial grid on the square are moved to new positions and are used to form the quadratic triangular grid that is colored in blue. Note that only the motions of the midpoints on the boundary of

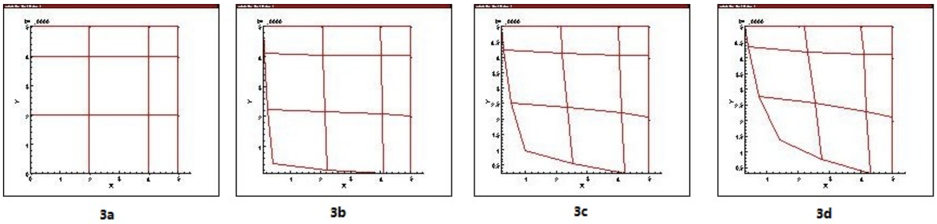



Fig. 3. Example 3

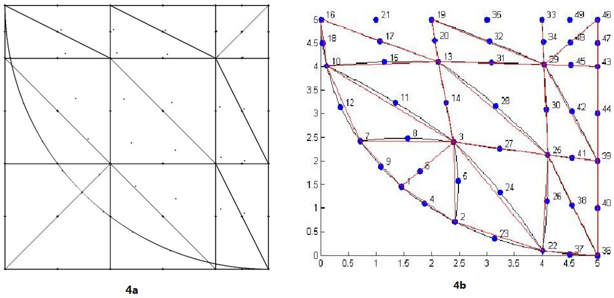



Fig. 4. Example 4

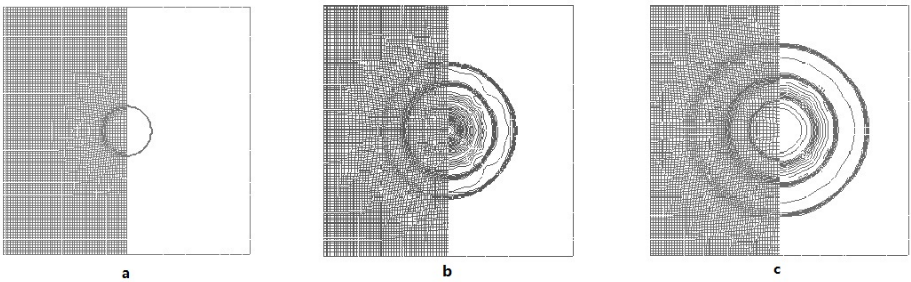



Fig. 5. Example 5

the initial square are specified. For instance (0, 1), (0, 3), (0, 4.5), (1, 0), (3, 0), (4.5, 0) are sent to the nearest points on the circle, respectively. See Figure 4b.

### *2.2. Solution-based adaptivity in CFD*

**Example 5:** Propagating Shock Fronts in [2]. In this example an unsteady compressible flow is formed with the initial condition of constant density $\rho_1$ inside a disk and $\rho_2$ outside the disk at $t = 0$, with $\rho_1 > \rho_2$. The discontinuity (shock front) propagates outward at $t > 0$. An initial grid adapted to the discontinuity is formed by the deformation scheme, which is dense near the circular boundary of the disk. As the shock front propagates, multiple expanding shock fronts are formed. Our deformation scheme redistributes the nodes so that the grid is always refined around the multiple shock fronts. The grids and the density contours at $t = 0, t = t_1$ *and* $t = t_2$ are shown on the left half and the right half of the Figures 5a, 5b, and 5c, respectively.

We observe that the nodes are not following the propagating fronts. In fact the grids are only refined near the fronts. This is a major advantage of redistribution strategy of grid refinement compared with the subdivision strategy, by which refinement is made by inserting nodes in a grid element that is located near a shock front. After the front moves to a new location, new nodes will be inserted and some nodes that are in a wrong location should be deleted.

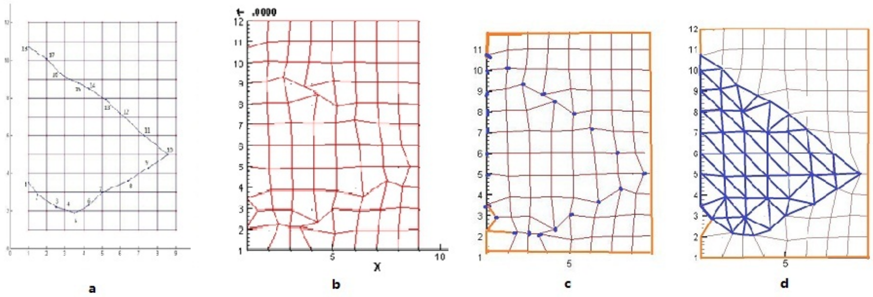


Fig. 6. Example 6

## 2.3. Deforming a cartesian grid to fit a curved domain

It was predicted by some leading experts [6] in CFD that Cartesian grids with AMR techniques will be widely used in production codes in the 2030s. But it is well known that treating small irregularly shaped cut cells on the curved boundary remains a very challenging problem. When boundary nodes are prescribed, Cartesian grid-based methods do not work according to [7]. We propose a new method which deforms a Cartesian grid to a body-fitted grid.

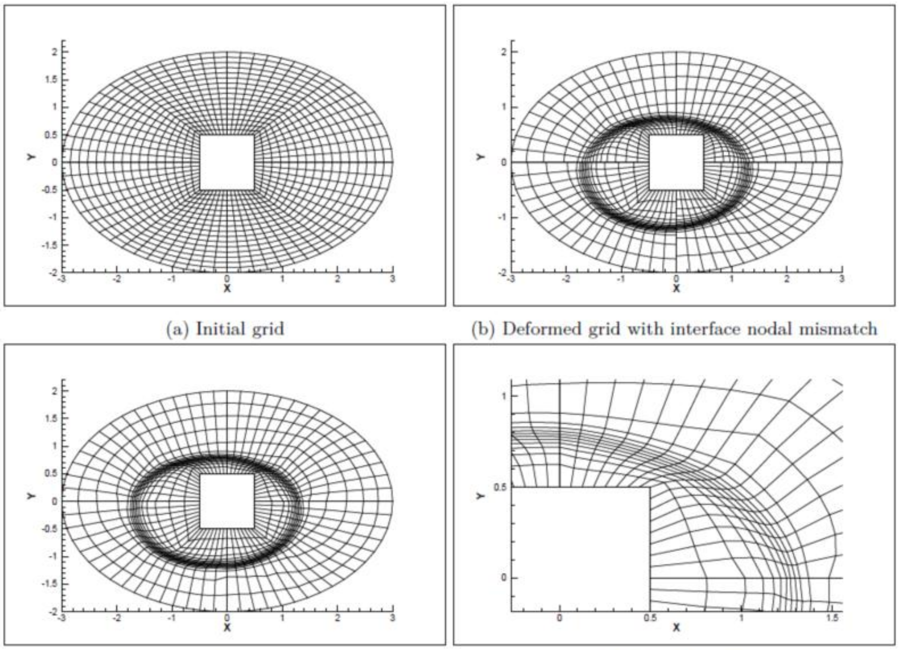
**Example 6:** Deforming a Cartesian Grid to Fit a Curved Domain with Prescribed Boundary Nodes. We demonstrate the feasibility of the proposed method on a domain D, which resembles the right half of the domain in Figure 8.2 in [7]. A uniform Cartesian grid on the rectangular domain [1,9] x [1,12] is shown in Figure 6a below. A domain D is described by points labeled from ♯1 to ♯18 on the curved boundary, and by the nodes $(1, 5), (1, 6), (1, 7), (1, 8), (1, 9)$ and $(1, 10)$ on the vertical boundary, see Figure 6a also. We then identify a set of 18 nodes on the Cartesian grid that are close to the boundary of domain D, and move them to the points ♯1 through ♯18, respectively. For instance, node $(2, 2)$ is moved to point ♯3; node $(4, 2)$ is moved to ♯5; node $(7, 4)$ is moved to point ♯9; node $(8, 5)$ is moved to ♯10; etc. Correct movements are enabled by imposing suitable Dirichlet condition on these nodes in the div-curl equations as if they were boundary points. The selected background nodes are shown in Figure 6b, which are moved to the prescribed boundary nodes ♯1 – ♯18, respectively. In Figure 6c, the background nodes are removed and the prescribed boundary nodes are colored in blue. Finally, a triangular grid is generated by connecting suitable pairs of the nodes on domain D, see Figure 6d. Reader may compare the triangular grid in Figure 6c with that in Figure 8.2 in [7].

## 2.4. Adaptive multiblock structured grids

It is well known that structured grids support accurate and efficient numerical solvers; but it is impossible to construct a single structured grid around an aircraft or even a wing of multiple airfoils. Therefore, we take the approach of multi-block structured grids which are generated in the following steps: (1) decomposing the complex domain into non-overlapping blocks; (2) generating a solution-based adaptive structured grid on each block (3) matching grids on interfaces of adjacent blocks to form an adaptive multi-block structured grid.

One of the leading commercial software grid generation products for producing multi-grid structured grids is GridPro [8]. Typically, structured grids are refined by node redistribution; unstructured grids are refined by local subdivision. GridPro is an interesting exception: the structured grids on selected blocks are globally subdivided and the refinement is kept in a compact region without propagating to far fields by clever positioning of singular nodes where adjacent blocks are joined singularly. This global strategy provides multi-level grids for the multi-grid methods. But the global subdivision strategy often results in too many nodes and too many blocks. This drawback is avoided in our deformation scheme. Example 7 shows that the proposed adaptive multiblock concept is promising.

**Example 7:** Adaptive Multiblock Grid. The domain D between an ellipse and a square is decomposed into four blocks by the coordinate axes, see Figure 7a. An initial O-shaped grid on D is generated. Then the grid on each block is deformed individually according to a monitor function. In general, the grids do not match on the interfaces, see Figure 7b. This mismatch is easily eliminated by the deformation scheme, which applies to pairs of adjacent blocks to match the grids on their interfaces, see Figures 7c and 7d (in a magnified view).



(a) Initial grid (b) Deformed grid with interface nodal mismatch

(c) Grid after treating mismatched nodes (d) Zoomed section

Fig. 7. Example 7

## 3. Ongoing and Future Activities

1. Develop a new grid deformation scheme that is capable of adapting grids according to a solution-based monitor function as well as to geometry changes; and explore the concept of making it a new stand-alone grid generation tool; apply this deformation method to generate high order adaptive unstructured grids and multi-block structured grids for 3D CFD calculation around aircraft;
2. Formulate a new optimization process based on deforming an initial grid according to geometry changes [5]. This process will be much more efficient since we only need to generate one grid of high quality; and the subsequent grids are obtained by adapting to the geometry updates.

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