

Research Note  
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# Anisotropic layering via curve insertion into unstructured meshes

Daniel W. Zaide\*, Carl F. Ollivier-Gooch

*Department of Mechanical Engineering, University of British Columbia, Vancouver, BC, Canada, V6T 1Z4*

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## Abstract

In this work, a new method for inserting anisotropic layers as internal boundaries into an existing mesh is developed. The anisotropic layers are inserted with minimal adjustment to the original topology while maintaining the original sizing of the mesh. The initial curve is sampled by optimizing point locations based on the underlying length scale of the initial mesh. The new layer locations are then determined by projecting the original sampling points in the normal direction, and the points are inserted to form quadrilateral cells within the offset layer. This algorithm allows for anisotropic layers to be added into existing meshes around areas of interest, such as material interfaces. The advantage of this algorithm over generating a new mesh is in preserving existing simulation data and reducing data interpolation onto the new mesh.

*Keywords:* Unstructured mesh adaptation, anisotropic meshing, curve insertion.

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## 1. Introduction

Often in the simulation of multi-physics or multi-materials, solid boundaries are determined after the initial simulation. Rather than generating a new mesh based on this boundary topology, we previously proposed an algorithm to insert these curves into the mesh with minimal mesh modification [1]. This work builds on that previous work, solving the problem of inserting several curves near each other, representing adding material interfaces as is often required in the simulation of semi-conductor devices. This method can also be used for mesh adaptation, creating structured anisotropic layers in regions of interest to capture physics or reduce numerical errors.

Our approach for layering is similar to that of Jain and Tautges [2] applied to triangular meshes if an existing boundary is present in the mesh. We generalize on this approach and do not require the presence of an initial existing boundary or other topological constraints provided that the offset locations do not intersect. We also utilize vertex removal in our method to ensure cell creation and reasonable spacing of points within the existing mesh.

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\* Corresponding author.  
*E-mail address:* cfd@mech.ubc.ca

## 2. Methodology

We begin with an existing unstructured triangular mesh and an initial curve and its offset curves, defined a small distance away from the initial curve. A length scale function on the mesh, denoted  $LS = LS(x, y)$ , is defined for vertices as a measure of the distance between neighbors on the mesh

$$LS(v) = \left( 2 \sum_{i=1}^N A_i \left/ \sum_{i=1}^N \theta_i \right. \right)^{\frac{1}{2}} \quad (1)$$

where  $N$  is the number of neighboring cells, with volume  $A_i$  and angle  $\theta_i$ . With this definition, the length scale at any point can be determined by barycentric interpolation within a cell.

Using the algorithm in [1], the vertices to insert on the curve are determined by equidistributing length scale along the curve based on an initial choice of points. The sampling is done on the initial curve, and the layers are generated by introducing additional sampling locations normal to the original curve on each successive layer. Once the curve locations between successive layers have been determined, we determine vertices in the original mesh to remove to make room for the new points to be inserted. Vertices that would either encroach on the new edge or that are within half a length scale of the new points are removed. To ensure we only check nearby vertices for removal, we only check vertices of cells containing sampling points.

## 3. Numerical Results

### 3.1. Example 1: Cubic spline on a uniform mesh

Consider the initial cubic spline defined by interpolation points  $(x, y) = (0.0, 0.5), (0.3, 0.4), (0.7, 0.6), (1.0, 0.5)$  to be inserted into a mesh defined on the unit square. We first insert it into the uniform mesh shown in Figure 1a. The initial mesh consists of 133 vertices and has a minimum angle of  $40.09^\circ$ . We will offset the cubic by a distance of 0.025 and create a quadrilateral layer in between. First, the curve is sampled based on the length scale of the initial mesh, with the 8 sampling points shown in Figure 1b. Next, we project these points normal to the curve to get the sampling points on the new curve, Figure 1c. Neighboring vertices are then checked, and the 16 vertices that would encroach on the new edges or are too close are flagged for removal in Figure 1d. Finally, the edges are recovered, 1e, and smoothing and swapping are used to return the mesh to a reasonable quality, with the final mesh in Figure 1f. In this example, the final mesh has 133 vertices and a minimum angle of  $28.19^\circ$ . On this mesh, 75% of the initial vertices have remained unchanged.

### 3.2. Example 2: Cubic spline on a graded mesh

Using the same geometry, we now demonstrate the algorithm on the graded mesh in Figure 2a. In this case, we'll insert five layers into the mesh below the curve, with a total offset distance of 0.01. The initial mesh has been created with a much smaller length scale on the right side of the mesh, and initially contains 274 vertices and a minimum angle of  $36.58^\circ$ . The curve is sampled, with 12 initial points in Figure 2a, leading to 72 total points to insert. With the sampling complete, 39 vertices are removed and the points are inserted. The final mesh in Figures 2b and 2c has 307 total vertices and a minimum angle of  $28.07^\circ$ . On the graded mesh, 70% of the initial vertices have remained unchanged and the layer insertion has not changed the grading of the mesh.

### 3.3. Example 3: Offset of an existing curve

Finally, to demonstrate the algorithm on a more relevant example, consider the 919 vertex mesh in Figure 3a, which contains an existing curve as an internal boundary. We will add three similar layers normal to this curve and create an anisotropic mesh, representing adding material layers in our simulation. The initial spacing is determined from the first set of edges, and all points to insert are in Figure 3b. After removing nearby vertices and inserting vertices, the final mesh in Figure 3c contains 934 vertices, and 84% of the initial vertices remain unchanged. The minimum angle, initially at  $30.41^\circ$ , is now  $29.64^\circ$ , demonstrating that our method maintains the quality of the original mesh.

## 4. Conclusions

In this work, we have developed and implemented an algorithm for anisotropic layer creation in an existing mesh based on our previous work on curve insertion. This algorithm chooses new point locations based on the length scale of the initial mesh, resulting in minimal additional work in the addition of these vertices to the mesh. With the new vertices inserted, existing mesh refinement and smoothing techniques are used to produce a final mesh with an anisotropic layer of quadrilaterals and an acceptable mesh quality. Future work will focus on the extension to three dimensions based on the three dimensional version of the previous work, resulting in layers of anisotropic prisms.

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## References

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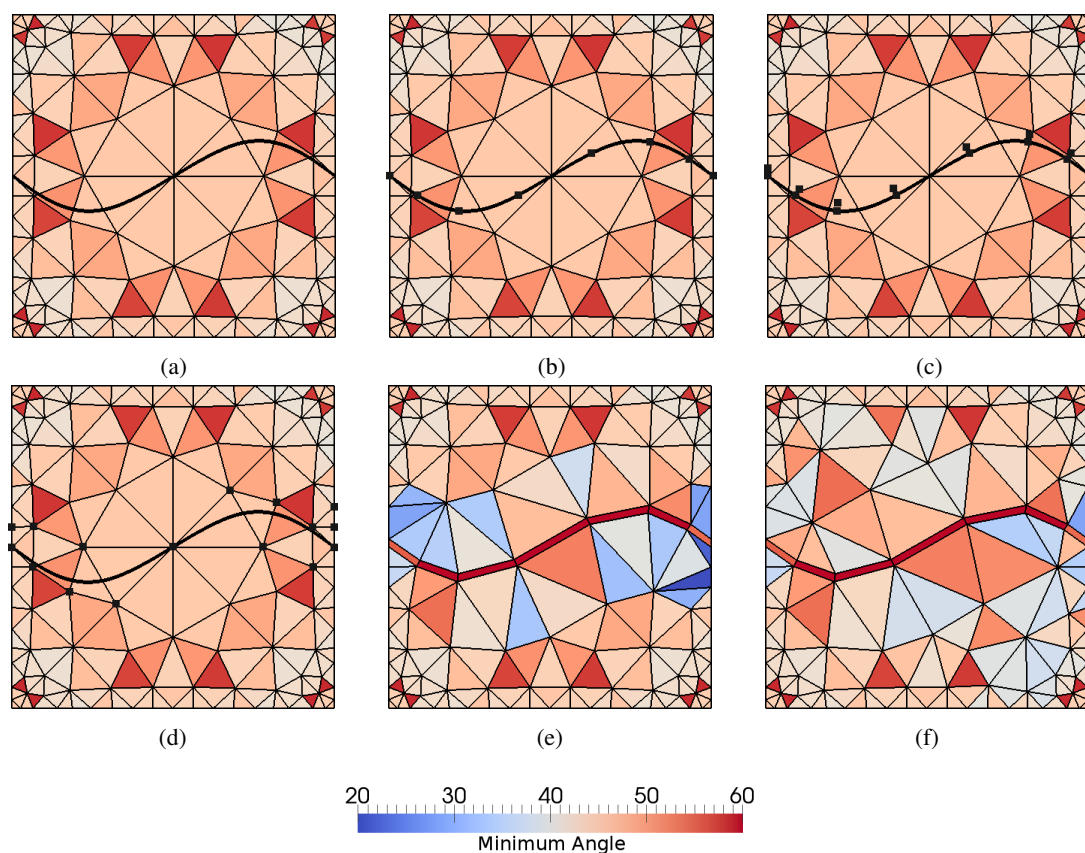


Fig. 1: Example 1: uniform mesh. Each cell is colored by minimum angle. a) Initial mesh, with curve. b) Initial points to insert. c) Offset points. d) Vertices to remove. e) Recovered quadrilateral layer. f) Final mesh, after local swapping and smoothing.

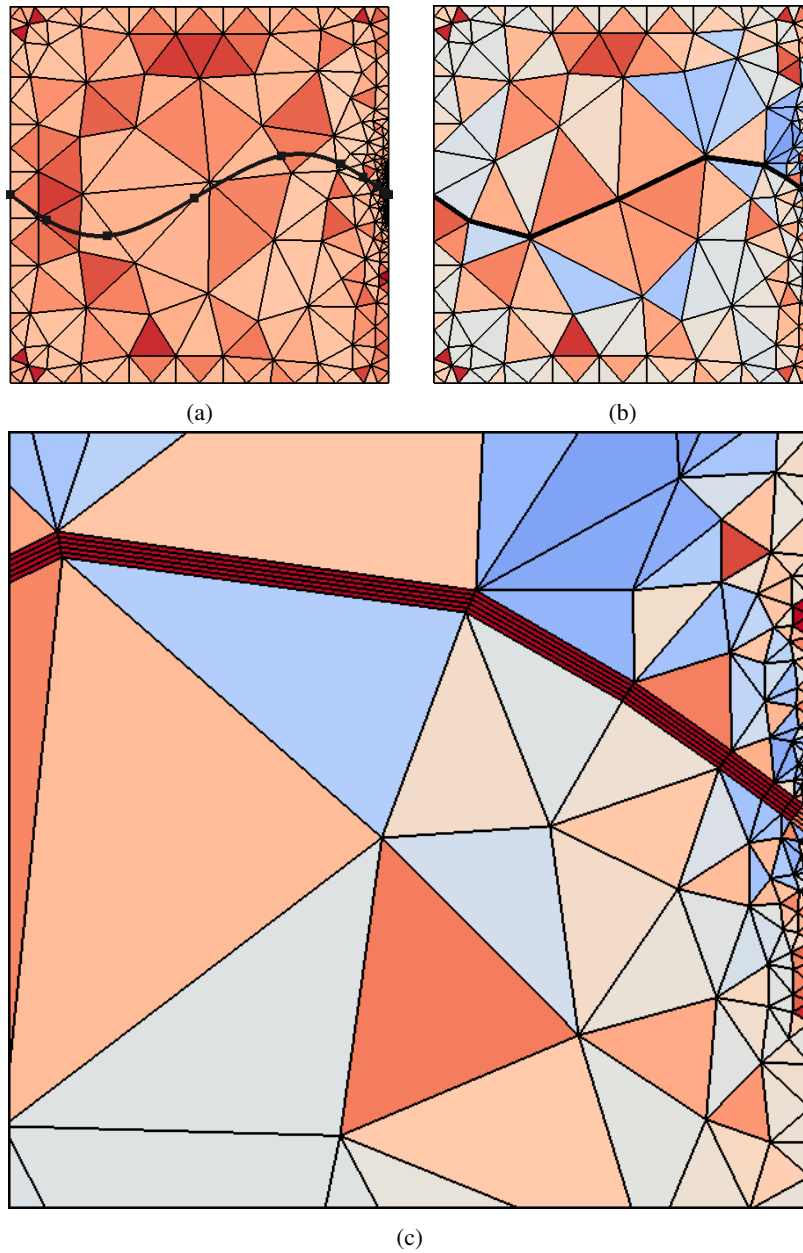


Fig. 2: Example 2: graded mesh. Five offset layers are created below the initial curve. Same coloring as Figure 1 a) Initial mesh, with points on initial curve to insert. b) Final mesh, after swapping and smoothing. c) Final mesh, zoomed in on the right side.

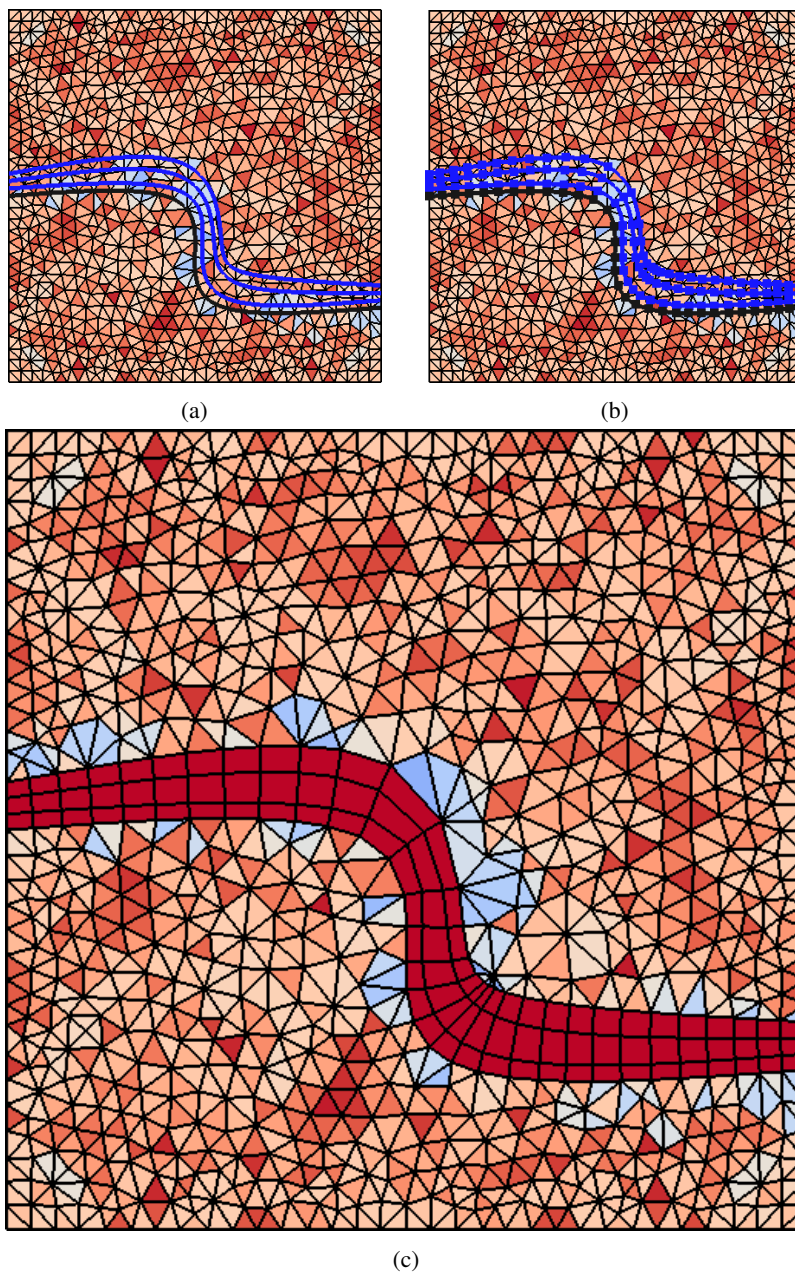


Fig. 3: Example 3: Quadrilateral layer formed by inserting additional, similar curves over an existing curve in the mesh. Same scaling as previous examples. a) Initial mesh with already inserted curve in black, and new curves in blue. b) Points to insert in blue computed from normals of initial curve. c) Final mesh.