NURBS Surface Reparameterization Using Truncated T-splines

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1 Introduction

Constructing T-splines for isogeometric analysis is an important step in integrating designing and analysis [1]. However, only a restricted type of T-splines is analysis-suitable, such as a subset of standard T-splines with constrains on the configuration of T-mesh [3]. Designed CAD models are not water-tight, and the control points are sparse. In addition, various operations result in trimmed NURBS patches, such as union, intersection, trimming and splitting (Fig. 1). These patches do not have tensor product representation in the trimming result. These limits the direct usage of CAD models for analysis. How to reparameterize designed CAD models with analysis-suitalbe T-splines is still an open problem.

To construct conformal T-spline surface from the designed CAD models, we propose an algorithm to reparameterize NURBS surface with truncated T-splines, which can be further used in building trimmed conformal T-spline surface and volumetric T-spline construction [6]. Truncated T-spline basis functions satisfy partition of unity for any T-junction insertion. Quadtree subdivision procedure is employed in the T-mesh construction, and we build a locally refined conformal truncated T-spline surface for the input NURBS surface.

2 Approach

Here we use truncated T-spines to reparameterize NURBS patches, which can then be used to generate water-tight T-spline surface. Our input is one NURBS patch with one trimming curve, and the output is a truncated T-spline representation conformal to the input NURBS patch with local refinement near the trimming curve.

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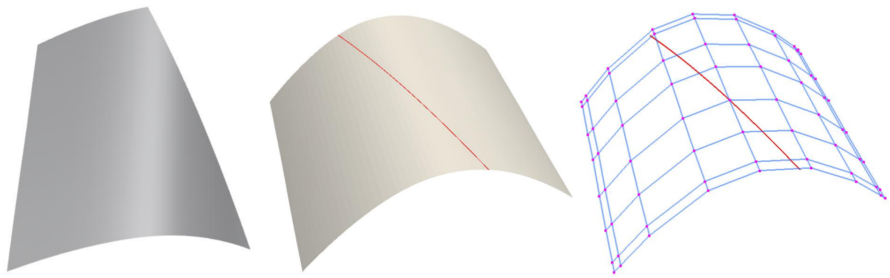


(a) (b) (c)

Fig. 1. Trimmed NURBS patch representation in IGES files. (a) Designed surface patch from trimming; (b) recalculated surface and trimming curve; and (c) the control mesh of the original surface with the trimming curve.

2.1 Truncated T-splines

T-spline basis functions are essentially B-spline basis functions defined on local knot vectors. Therefore they inherit the refinability property from B-spline basis functions. Consider a local knot vector on which a T-spline basis function $N_i(u,v)$ is defined, we obtain new local knot vectors by inserting one or multiple knots into the original knot vector. Refinability indicates that $N_i(u,v)$ can be represented by a linear combination of $N_j^c(u,v)$ defined on these new local knot vectors. We have

$$N_i(u,v) = \sum_j c_{ij} N_j^c(u,v), \tag{1}$$

where c_{ij} are the coefficients obtained from knot insertion algorithm and $N_j^c(u,v)$ are called *children* of $N_i(u,v)$. The truncation of $N_i(u,v)$ is performed by discarding its certain identified children in Eqn. 1 [2, 5]. The identification of the children to be discarded will be discussed later in Section 2.2. The truncated T-spline basis functions form a partition of unity and have more compact support, which play a significant role to preserve the geometry during local refinement.

A truncated T-spline surface is defined as

$$S(u,v) = \frac{\sum_{i=0}^{n} P_i N_i^t(u,v)}{\sum_{i=0}^{n} N_i^t(u,v)}, \tag{2}$$

where P_i are $n+1$ control points and N_i^t are truncated T-spline basis functions. Note that all the weights for control points are set to be 1. For a T-mesh with general configuration, its basis functions may not satisfy partition of unity. However, our truncated T-spline basis functions ensure that $\sum_{i=0}^{n} N_i^t(u,v) = 1$

holds everywhere. The truncated T-spline control mesh is generated from subdivision and supports any type of T-junction configuration.

2.2 Subdivision Procedure

We compute n equally distributed sampling points on the trimming curve and obtain their approximated parametric coordinates on the surface. These sampling points are used to decide the local refinement region.

Subdivision. The original NURBS control mesh is set as the initial T-mesh. We perform strongly balanced local quadtree subdivision to the T-mesh in the parametric domain. If the new T-spline surface is conformal to the input, we have $\sum_{i=0}^{m} N_i(u,v) = \sum_{j=0}^{n} N_j^t(u,v) = 1$, and

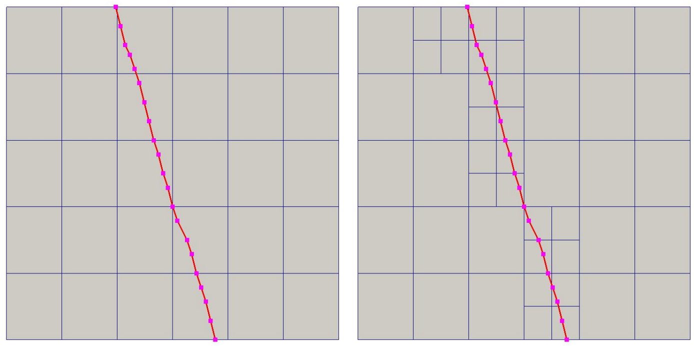
$$\begin{aligned} S(u,v) &= \sum_{i=0}^{m} P_i N_i(u,v) = \sum_{j=0}^{n} P_j^s N_j^t(u,v) \\ &= \sum_{j=0}^{n} P_j^s (N_j^s(u,v) - \sum_{k} h_{jk} N_k^c(u,v)), \end{aligned} \tag{3}$$

where P_j^s and N_j^t are control points and associated truncated basis functions after local subdivision, $N_j^s(u,v)$ are T-spline basis functions defined on the T-mesh after subdivision, and $N_j^c(u,v)$ are the to-be-discarded children with coefficients h_{jk}. Since the original T-spline space is a subspace of the refined space [4], each basis function N_i can be represented as a linear combination of N_j^t via the transformation matrix obtained from the transformation $\mathbf{N} = \mathbf{M}\mathbf{N^t}$, where $\mathbf{N}$ and $\mathbf{N^t}$ are the matrices formed by basis functions N_i and N_j^t, respectively. $\mathbf{M}$ is also used to calculate P_j^s.

Basis Truncation. For one control point, we first insert knots to make its knot vector fully refined. For example, the knot vector $\{0, 1, 2, 3, 3.5\}$ is refined as $\{0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5\}$. With Eqn. 1, before subdivision we have $\sum_{i=0}^{m} N_i(u,v) = \sum_{i=0}^{m} \sum_{k} c_{ik} N_k^c(u,v) = 1$ holds for any (u,v). After subdivision, we have $\sum_{j=0}^{n} N_j^s(u,v) = \sum_{j=0}^{n} \sum_{k} c'_{jk} N_k^c(u,v)$. We discard $\Delta = \sum_{j=0}^{n} \sum_{k} c'_{jk} N_k^c(u,v) - \sum_{i=0}^{m} \sum_{k} c_{ik} N_k^c(u,v)$ from $\sum_{j=0}^{n} N_j^s(u,v)$. For one basis function $N_j^s(u,v)$, its corresponding coefficient is decided by its contribution to Δ with

$$h_{jk} = \frac{c'_{jk}}{\sum_{j=0}^{n} \sum_{k} c'_{jk}} (\sum_{j=0}^{n} \sum_{k} c'_{jk} - \sum_{i=0}^{m} \sum_{k} c_{ik}). \tag{4}$$

The subdivision continues if there is more than one sampling point falling in the same T-mesh element, or the T-mesh is not strongly balanced. Fig.



(a) (b)

Fig. 2. Reparameterization with truncated T-spline surface for the trimmed NURBS patch in Fig. 1. (a) The parametric domain of the patch and the approximated sampling points of the trimming curve; and (b) iteratively subdividing the NURBS patch and introducing new T-junctions near the trimming curve in the parametric domain.

2 shows the first level of subdivision of the patch given in Fig. 1(b). With certain level of subdivision, we can approximate the trimming curve on the model. Higher level of subdivision will result in better approximation result.

3 Results and Discussion

One preliminary result of the NURBS surface is shown in Fig. 3, where only one level of subdivision is performed. The input NURBS patch in Fig. 1(b) is reparameterized with both standard T-spline and truncated T-spline. The truncated T-spline surface is conformal to the input. Comparison with standard T-spline shows that our method has less control points for the same level of subdivision. More efforts will be put in the future to make our code work for T-mesh with any subdivision level.

In conclusion, we proposed one algorithm to construct conformal truncated T-spline from NURBS surface with trimming curve constrains. The resulting spline surface is exactly the same as the input and satisfy the partition of unity property. The presented algorithm can be further used for analysis-suitable conformal volumetric T-spline construction. In the future, we plan to perform trimming operation to the T-spline surface, and construct water-tight truncated T-spline surfaces with multiple patches from designed CAD models.

Acknowledgements

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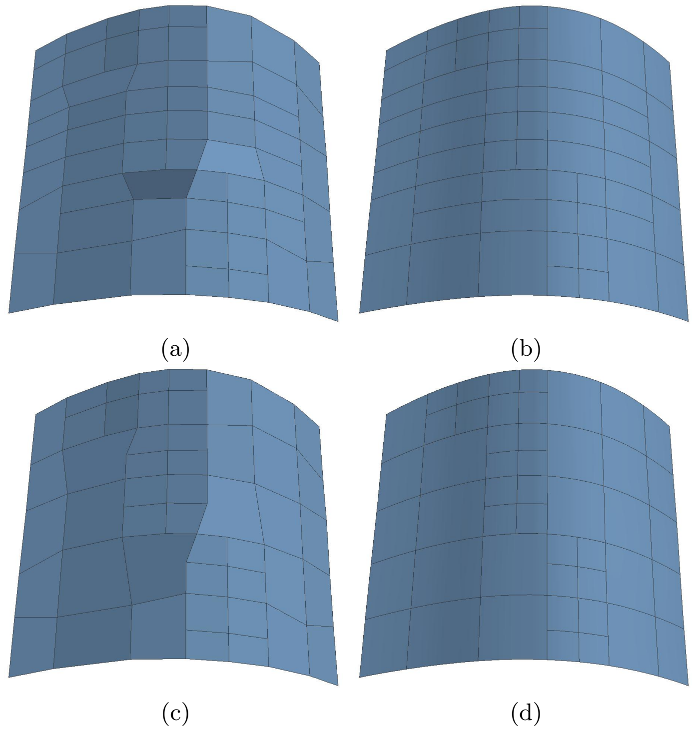

Fig. 3. Results of conformal T-spline surface with one level subdivision. (a) T-mesh of standard T-spline; (b) calculated standard T-spline surface; (c) T-mesh of truncated T-spline; and (d) calculated truncated T-spline surface.

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