Using Edge Valence Prediction to Drive Localized All-Hex Coarsening

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Abstract: In this work, we propose using edge valence as a quality predictor when used as a driver for adapting all hexahedral meshes. Edge valence, for hexahedra, is defined as the number of faces attached to an edge. It has shown to be a more reliable quality predictor than node valence for hexahedral meshes. This work presents a general algorithm for predicting edge valence when used with column collapse and sheet extraction operations without actually performing the operation. Results have been derived from application of the algorithm towards the localized coarsening process.

Keywords: Hexahedra, Coarsening, Mesh Topology Modification, Edge Valence

1 Introduction

Several mesh adaptation algorithms exist for hexahedral meshes including coarsening [4], refinement [1], mesh matching [3], and grafting [2]. An existing mesh may be modified to adjust node density through refining and/or coarsening to increase the quality of elements of a mesh, to create a conformal mesh, or to more easily mesh a domain.

The purpose of this work is to develop a method for guiding the localized hexahedral coarsening process [4] based on a reliable quality metric. This method must accurately predict mesh quality for sheet operations without actually performing those operations in order to guide the modification process. Sheet operations of importance to this work include column collapse, sheet extraction, and pillowing.

Edge valence is a relatively new quality metric developed by Staten [6] that has shown to be an accurate predictor of hexahedral mesh quality. Edge valence is defined as the number of quadrilateral faces connected to a single edge. Staten asserts that if the edge valence of all edges in a hexahedra mesh is 3, 4, or 5 then the scaled Jacobian of that element will be greater than zero and likely much higher, in the absence of over constraining geometric topology. For an edge on the interior of the mesh, if the edge valence is less than 3 the element will contain a doublet and consequently be inverted. Elements with an edge valence greater than 5 may have acceptable quality but if a doublet exists, the element will be inverted and admit only poor quality.

2 Edge Valence Prediction

A new algorithm that can be used during hexahedral coarsening is presented by the author in [5]. This algorithm accurately predicts the valence of each edge in a given mesh through the steps of column collapse and sheet extraction. The algorithm assumes the coarsening region has been selected, the pillow has been inserted, and a coarsening layout has been determined. For the purpose of this study, a coarsening layout is defined as a set of columns and sheets that could be, respectively, collapsed and extracted to produce a coarsened mesh.

For this algorithm equation 1 is used to predict the edge valence for edges that will merge with another edge during column collapse and equation 2 is used for edges that will not merge with another edge during column collapse. Equation 3 is used to predict edge valence for edges during sheet extraction. These equations are presented with reference to the examples in figures 1 and 2

$$PEVC = m_1 + m_2 - 2m_3 + a \tag{1}$$

$$PEVC = m - 1 + b \tag{2}$$

where:

PEVC = Predicted edge valence for column collapse

m_1= Number of hexes attached to edge 1

m_2= Number of hexes attached to edge 2

m_3=Number of hexes common to edge 1, edge 2, and the column

m = Number of hexes attached to edge

a = 1 if at least 1 edge is on the mesh boundary, 0 otherwise

b = 1 if edge is on mesh boundary, 0 otherwise

$$PEVS = m_1 + m_2 - 2m_3 + a \tag{3}$$

where:

PEVS = Predicted edge valence for sheet extraction

m_1 = Number of hexes attached to edge 1

m_2 = Number of hexes attached to edge 2

m_3 = Number of hexes common to edge 1, edge 2, and the sheet

$a = 0$ if neither edge is on the mesh boundary, 1 otherwise

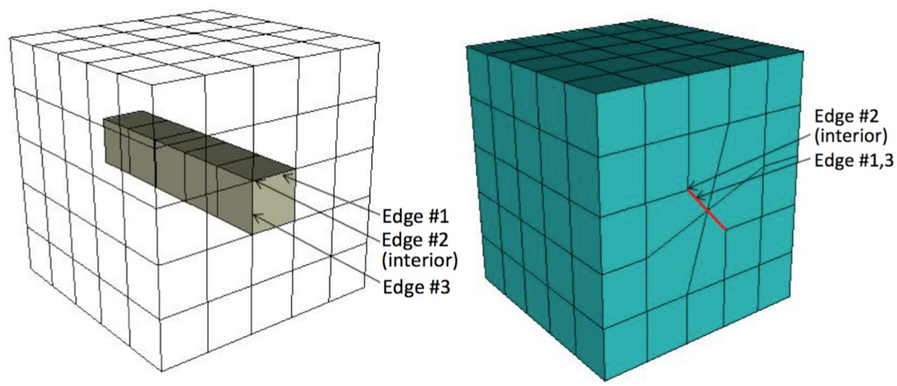

Fig. 1. Column collapse operation

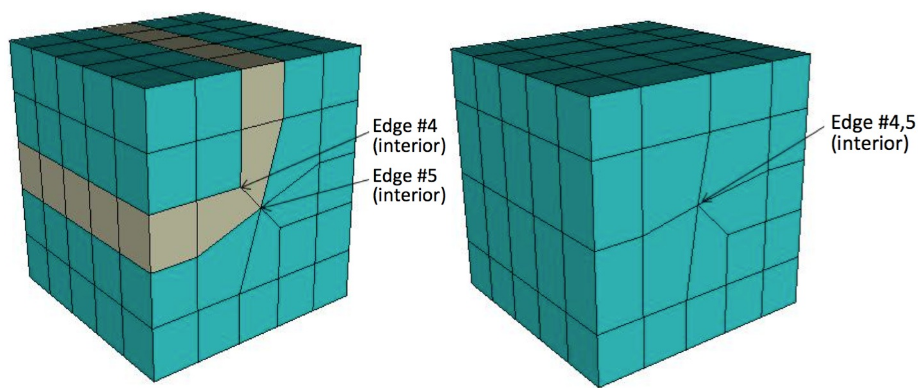

Fig. 2. Sheet extraction operation

The equations presented above were used to predict edge valence prior to performing collapse or extract operations. These can be used to avoid creating conditions where an edge valence less than three and greater than five will be created.

3 Results

The following example demonstrates the ability of the edge valence prediction algorithm to accurately predict edge valence through sheet operations as well as its ability to guide the coarsening process. Figure 3 shows an initial mesh in four volumes with a coarsening zone shown highlighted. The coarsening zone extends five layers into the mesh. It also shows the resulting meshes after 66 per cent coarsening. Table 1 summarizes the results comparing predicted and actual edge valence. This table also shows the number of 3 through 7 valent edges in each coarsened region. Figure 4 shows a histogram of the scaled Jacobian comparing with and without edge valence prediction.

4 Conclusions

This research note demonstrates the application of edge valence prediction for all hexahedral mesh coarsening. This extends the coarsening algorithm in [4] to help guide the procedure resulting in improved overall mesh quality. Demonstration of the ability to guide mesh modifications without actually altering the mesh is the principle contribution of this study.

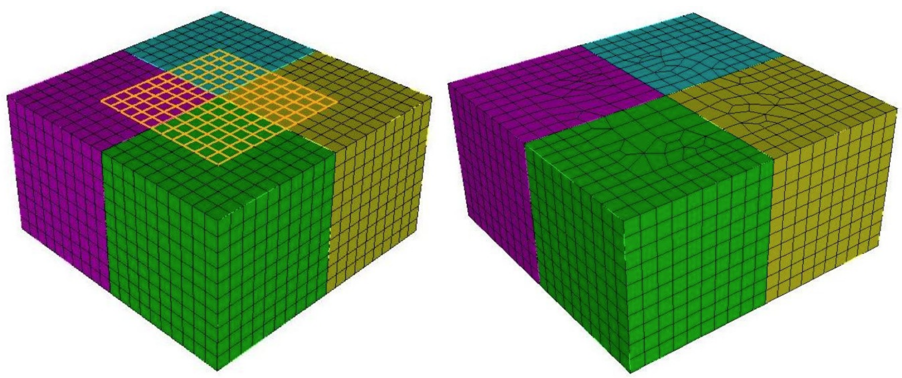

Fig. 3. Left: Simple block model mesh where highlighted region to be coarsened extends 5 layers. Right: Resulting coarsened mesh with target 66 percent coarsening

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Table 1. Edge valence and scaled Jacobian results for coarsening of block model

	Without Prediction	With Prediction	
Min Scaled Jacobian	0.209	0.431	
Avg Scaled Jacobian	0.742	0.871	
Volume Edge Valence	Edges Produced	Edges Predicted	Edges Produced
3	294	172	172
4	8046	8492	8492
5	247	188	188
6	35	0	0
7	3	0	0
Surface/Curve Edge Valence	Edges Produced	Edges Predicted	Edges Produced
2	200	200	200
3	2912	2920	2920
4	616	626	626

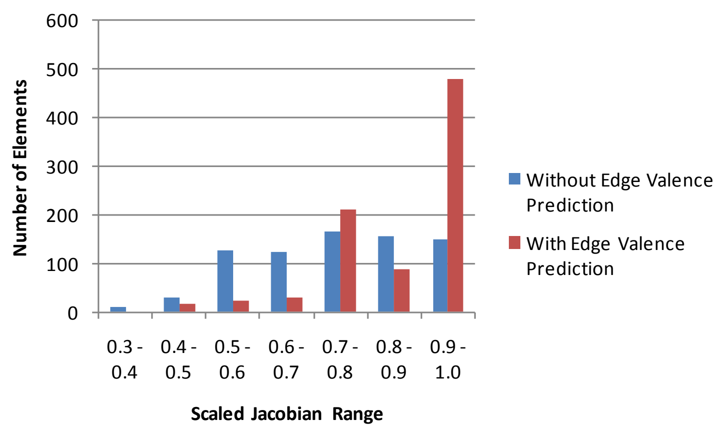

Fig. 4. Histogram of scaled Jacobian for coarsened block model comparing coarsening results with and without edge valence prediction

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