Adaptive anisotropic meshing and remeshing technique for the numerical modelling of SAGD process

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1 Introduction

This note describes an adaptive meshing and automatic remeshing technique for the numerical modelling of the Steam Assisted Gravity Drainage process and called SAGD. The topics will be illustrated here is the numerical modelling of the development of steam chamber in the reservoir field. Steam chamber inevitably occurs during thermal and geomechanical processes and have a major impact on the final oil recovery. This phenomena seems to be simple but their mechanisms are not very well understood. Different methods allow to model SAGD process and a central idea realized through several studies is that the near-well region must be heated rapidly and efficiently for significant early-time response. To improve early-time response of SAGD, it is necessary to heat the near-wellbore area to reduce oil viscosity and allow gravity drainage to take place. Ideally heating should occur with minimal circulation or bypassing of steam. Since project economics are sensitive to early production response, we are interested in optimizing the start-up procedure. An investigation of early-time processes to improve reservoir heating will be discussed. We perform a numerical simulation study of combinations of steam injection and steam circulation prior to SAGD in an effort to better understand and improve early-time response. In these case, the quality of the numerical solutions are strongly related to the accuracy of calculations around the considered temperature front, and so for a given computational cost, optimal accuracy is obtained by using anisotropic meshes, with refinement close to the high error of temperature field. Periodic remeshing is performed such as the refinement zone describe accurately the temperature front displacement. Results from this study, including temperature distributions, display the behaviour of steam diffusion and conduction prior to SAGD. The mesh adaptation improve dramatically the description of the temperature distributions.

2 Anisotropic metric field based on error estimator

In order to construct an anisotropic mesh, an appropriate anisotropic metric has to be defined. In our approach, the construction of the metric is based on the minimization of a error estimator model. The error model is defined as a function of the hessian of a the solution u_h. A minimisation problem is then derived to minimize this function under a fix number of elements constraint. As introduced in our previous work [4], the upper bound of the local error estimator is given by the following expression:

$$\eta_T \leq d|T|^{\frac{1}{p}} |\lambda_d(x_0)| h_d^2 \tag{1}$$

This shows that the local estimator is bounded by the maximum of the second order derivative in the barycenter of the element times the square of the longitude in this direction.
We may introduce as the local estimator the above upper bound and is given by

$$\eta_T \;=\; d|T|^{\frac{1}{p}} |\lambda_d(x_0)| h_d^2 \tag{2}$$

Let $\mathcal{T}_h$ denote the current finite element discretization of the domain Ω, u_h denotes the approximate solution associated to the mesh $\mathcal{T}_h$, $h_{old}(P)$ describes the local mesh size at node P in the direction of the maximum value of the directional second order derivative at the point P. Then, given the number of desired elements $N_{\mathcal{T}'_h}$ in the new adapted mesh, the optimal mesh adaptive procedure generates a *new* mesh, $\mathcal{T}'_h$, such that the new distribution $h_{new}(P)$, for all $P \in \mathcal{T}_h$ minimizes the global estimator error. Hence, the optimal mesh adaptive procedure looks for an optimal mesh as a solution of the following constrained optimization problem

$$\begin{cases} \text{Find } h_T = \{h_{1T}, \cdots, h_{dT}\}, T \in \mathcal{T}_h \text{ that minimizes the cost function} \\[2ex] F(h_T) = \displaystyle\sum_{T \in \mathcal{T}_h} \left(\eta_T\ \right)^p \\[4ex] \text{under the constraint} \quad N_{\mathcal{T}'_h} = \; C_0^{-1} \displaystyle\sum_{T \in \mathcal{T}_h} \int_T \prod_{i=1}^{d} \frac{1}{h_{iT}}\, dT \end{cases} \tag{3}$$

where C_0 is the volume of a regular tetrahedron. The optimization problem (3) was studied in the case of Multi-dimensional unstructured meshes in [4]. The following theorem summarizes the anisotropic error estimator proposed in [4].

Theorem 1. *For $d = 3$, the optimization problem (3) has a unique solution and is given by*

$$\begin{cases} h_{3T} = \left[\quad \dfrac{\beta}{\frac{(2p+3)}{3} \; C_{1T}} \int_T C_{2T} \; dT \quad \right]^{\frac{1}{2(p+3)}} \\ \\ h_{2T} = s_{2T} h_{3T} \\ \\ h_{1T} = s_{1T} s_{2T} h_{3T} \end{cases} \tag{4}$$

where

$$C_{1T} = 3^p C_0 s_{1T} s_{2T}^2 |\lambda_3|^p, \qquad C_{2T} = C_0^{-1} \frac{1}{s_{1T} s_{2T}^2} \qquad \textit{and}$$

$$\beta^{\frac{1}{\frac{2}{3}(p+3)}} = N_{\mathcal{T}_h'}^{-1} \sum_{T \in \mathcal{T}_h} \Big\{ \Big(\frac{1}{\int_T C_{2T} \; dT} \Big)^{\frac{1}{\frac{2}{3}(p+3)}} \int_T C_{2T} \Big[\; \frac{2p+3}{3} C_{1T} \; \Big]^{\frac{1}{\frac{2}{3}(p+3)}} \; dT \Big\}.$$

In what concerns re-meshing, our algorithm implies that around an arbitrary point P of the mesh, we try to build equilateral tetrahedrons in the metric defined by the local metric field $\mathcal{M}$, according to a local topological technique. This metric is defined in R^d by:

$$\mathcal{M}(P) = \frac{1}{h_1(P)} e_1 \otimes e_1 + \cdots + \frac{1}{h_d(P)} e_d \otimes e_d \tag{5}$$

where $(e_i)_{i=1,d}$ are the eigenvectors of the recovered hessian $H(u_h(P))$ and $h_i(P)$ are the optimal mesh sizes in the e_i directions.

3 Anisotropic mesh adaptation algorithm

We know now the optimal distribution of the elements shapes and the stretching directions that are given by the eigenvectors of the recovered Hessian as well as the optimal metric that will be used with the background mesh. Both are used as an input of the mesh generator tool, called MTC, in order to obtain a new (optimal) mesh. The mesh generator used here and its parallelization are described in [6] and [5]. MTC is a mesh generator developed by Thierry Coupez at the Ecole des Mines de Paris, Center for Material Forming, Sophia Antipolis. It is based on the idea to improve iteratively, an initial unsatisfactory mesh by local improvements.

MTC re-meshes the initial mesh iteratively by a local mesh optimization technique. This technique consists in local re-meshing of cavities formed by small clusters of elements in order to increase the “quality” of the elements of the cluster.

4 The steam assisted gravity drainage process

SAGD is a thermal in-situ heavy oil recovery process. The procedure is applied to multiple well pairs. The well pairs are drilled horizontal, parallel and vertically aligned with each other; their length and vertical separation are on the order of 1 kilometer and 5 meters, respectively. The upper well is known as the “injection well” and the lower well is known as the “production well”. The process begins by circulating steam in the injection well so that the bitumen between the well pair is heated enough to flow to the lower production well. The freed pore space is continually filled with steam forming a “steam chamber”. The steam chamber heats and drains more and more bitumen until it has overtaken the oil-bearing pores between the well pair. The cone shaped steam chamber now begins to develop upwards from the injection well. As new bitumen surfaces are heated, the oil lowers in viscosity and flows downward along the steam chamber boundary into the production well by way of gravity. Figure 1.2 illustrates the concept with a typical well pair. A central idea realized through several studies is that the near-well region must be heated rapidly and efficiently for significant early-time response.

To improve early-time response of SAGD, it is necessary to heat the near-wellbore area to reduce oil viscosity and allow gravity drainage to take place. Ideally heating should occur with minimal circulation or bypassing of steam. Since project economics are sensitive to early production response, we are interested in optimizing the start-up procedure. We perform a numerical simulation study of combinations of steam injection and steam circulation prior to SAGD in an effort to better understand and improve early-time response. Results from this study, including temperature distributions, display the behaviour of steam diffusion and conduction prior to SAGD are given in section 5. The mesh adaptation improve dramatically the description of the temperature distributions.

5 3D SAGD modeling

Numerical testing is performed using finite element simulations where two models are considered: Incompressible Navier-Stokes and heat transfer. The flow is supposed in a mixed convection regime and an outflow region, leading to a coupled problem between the Navier-Stokes and enrgy equations. The mixed finite element discretization chosen here is computationally efficient. The objective of the test case is to analyze the distribution of temperature in the reservoir, subjected to the development of a steam chamber. Figure **??** shows the distribution of temperature. In this example, boundary layers are assumed to be due to the difference in temperature gradient on either side of the well. When dealing with primary SAGD process, bondary layers need to be modelled as well. Figure 2 illustrates the evolution of the injected steam with automatic adapted remeshing operations.

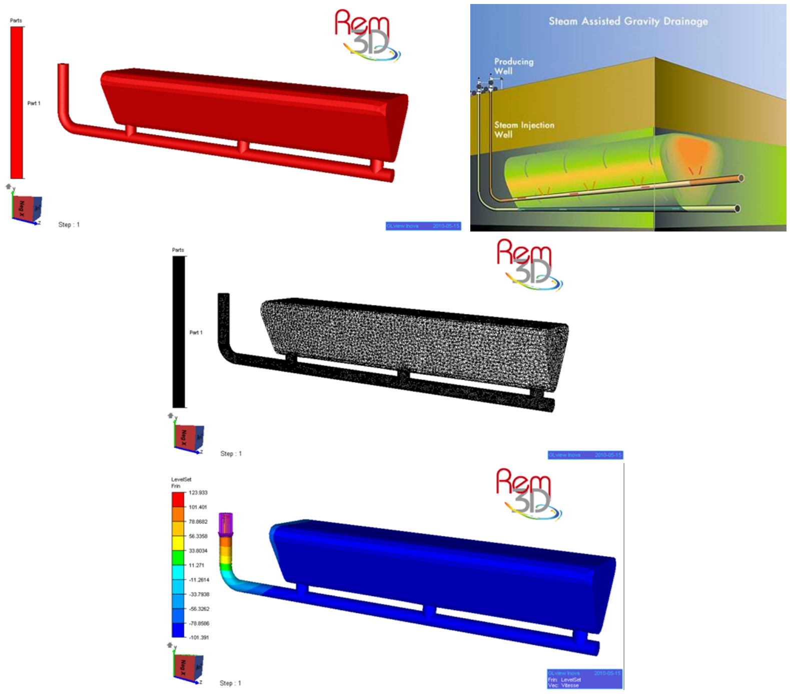

Fig. 1. SAGD CAD, configuration, mesh and point injection

Figure 4 an Figure 5 illustrates the same configuration in 3D with different increment of time.

6 Conclusion

It has been shown that an adaptive anisotropic automatic remeshing based on error minimization is a promising tool to describe a steam chamber development in a Steam Assisted Gravity Drainage process. Indeed, for SAGD modelling, heat front can be more described, and accurate solutions can be derived with a reasonable CPU time. In this topic, various studies are in process, including simulations of full reservoir heating, the second step of the process that is the drainage stage and then the last and not least one the pumping.

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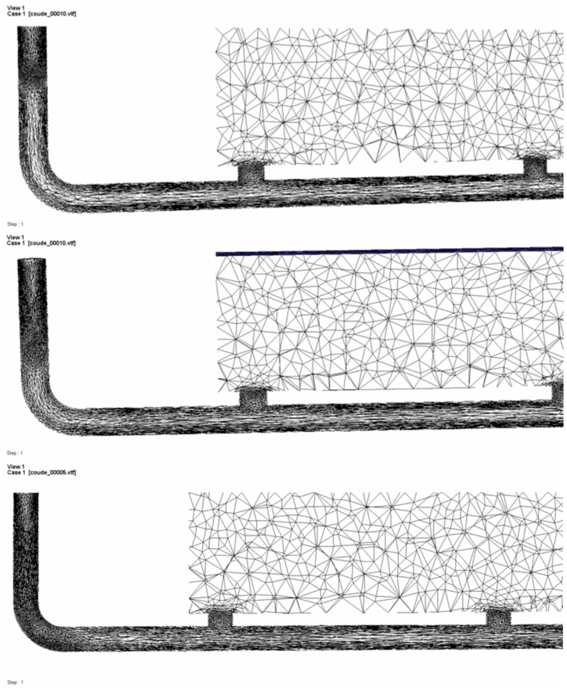

Fig. 2. Evolution of the mesh with respect to temperature calculations

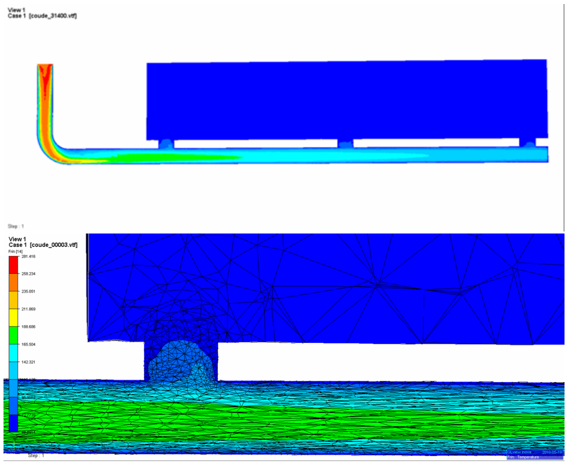

Fig. 3. Evolution of the temperature distribution

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