Graph Based Boundary Recovery for Complex Geometry with Multiply Connected Domains

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Abstract.

The paper addresses the problem of the 3D meshing of the complex free form Boundary Representation geometry with multiple domains. It introduces a generic graph based method of the CAD model analyses with optimal boundary recovery. A three stage approach is used: firstly, an underlying geometry is repaired to provide a conformal model without gaps and overlaps. Secondly, the model is analyzed and a special topological graph is introduced. Finally, an unstructured 3D Delaunay mesh is generated using the properties of the spanning tree of the domain graph to provide an optimal boundary recovery during meshing with a minimal number of inserted nodes. The properties of the domain graph are discussed in the context of unstructured mesh generation. The paper provides examples, showing efficiency of the method for optimal meshing of complex Geological Geometry.

1. Introduction

In recent years the development of robust mesh generation and CAD healing techniques [1] has significantly increased the complexity of meshed geometry. Now it is possible to perform 3D unstructured meshing of different CAD models - Constructive Solid and Boundary Representation (BREP) - with multiple sub-domains in different application areas. Moreover, recent developments have extended applicability of the CAD solid models from traditional engineering domains to natural sciences, such as geology, reservoir engineering, and hydrology. Meshing of such complex geometry (see, for example, Fig. 1) faces challenges of efficient boundary recovery and the problem is mostly related to the absence of a generic methodology of the CAD model analyses for meshing [1].

The presence of multiple geometrical constraints in the CAD model (Fig. 1) results in insertion of excessive nodes during boundary recovery in the traditional meshing approaches similar to [2]. All-hexahedral meshing faces more difficulties, related to the generic complexity of the unstructured hexahedral meshing algorithms for free form domains. The Control Volume (CV) node-centered discretisation is also non-optimal [3-4], as near the boundary a node (see Fig. 2) is typically shared by a large number of elements, and the CV flux computation for the node-centered schemes involves large numbers of the CV cell polyhedral faces, as shown in Fig. 2. This significantly slows the computation [4].

Abstraction of the BREP models has been addressed in different spheres of engineering. For example, Similarity Analyses of the CAD models provide an interesting background for the CAD model analyses for meshing (see [5] for an overview of the methods). Shape abstraction techniques summarize topological properties of the CAD model partition of space to sub-volumes.

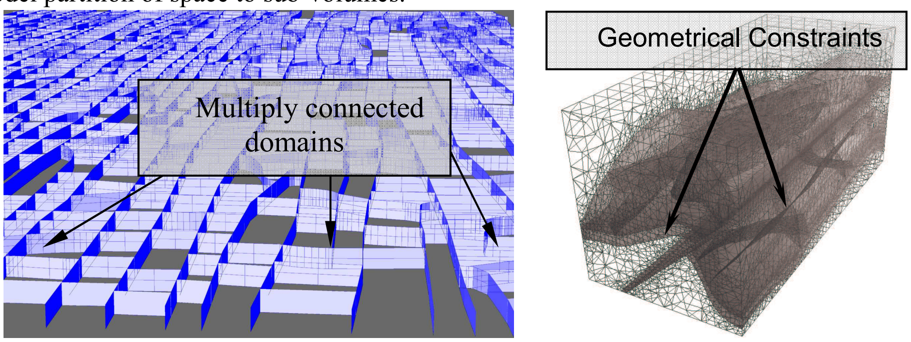
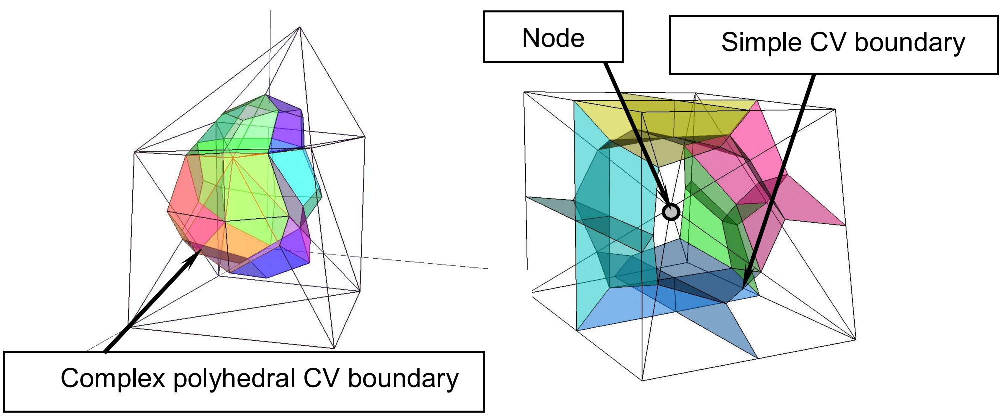


Fig. 1. Typical geological BREP geometry with multiple (N>100) domains: the CAD model (fractured stone) and hex-dominant mesh of the fault. Numerous CAD constraints make the boundary recovery difficult and induce large number of nodes and elements.



a) b)

Fig. 2. The node centered CV polyhedral cell: a) with a complex (38 facets) boundary for all-tetrahedral and b) boundary-optimal (20 facets) hybrid cell.

A conformal BREP CAD model provides a 3D partition of space to free-form sub-domains, further CAD meshing ensures domains subdivision to computational primitives – tetrahedrons, hexahedrons, etc. So far systematic connection of the CAD partition with computational meshing has not been established and our work attempts to close this gap using the graph-theoretical approach.

2. Proposed Method

The idea of the method is based on application of the Shape Similarity Assessment Algorithms [5] to the underlying BREP model. Similar methods are used in the area of product design and manufacturing. In this area, a number of efficient CAD model abstractions have been developed to define a measure of CAD model similarity for production purposes. One of the measures is using topological graphs of the solid CAD model to provide a summary of the object shape for downstream applications, i.e. for mesh generation. In our implementation the method uses a direct interface to the BREP CAD model, when a solid is defined by a special data structure [6], providing the geometry and topology of its bounding faces. Recently the BREP approach has become the representation of choice in solid modeling for engineering and natural science applications due to its flexibility and robustness [4,6].

Previously graph approach have been used in mesh generation for hybrid meshing [7], mesh optimization [2] and other problems, taking advantage of the efficiency and robustness of the graph based algorithms. However, diversity and complexity of existing CAD models still pose a challenge for an efficient CAD analyses for meshing.

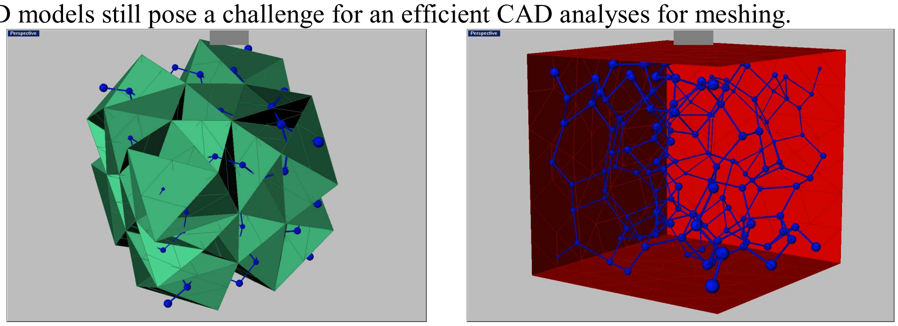


Fig. 3. The MSG graph (the blue nodes and edges, left and right) for a synthetic cube geometry with multiply connected domains, partitioned by green free form surfaces (left).

Topological graphs represent the connectivity information of the solid boundary such as adjacency of faces. Meshkat and Talmor [7] used graphs, representing adjacency of volumes (region-face graph technique) for hybrid meshing, but it is hardly applicable to meshing-related CAD model analyses. Indeed, the techniques, proposed in [7] provide the formalism for hexahedra and pentahedra composition from existing tetrahedrons. However, the so called Model Signature Graph (MSG) is capable of providing useful information on geometric constraints of the CAD model. The MSG is directly constructed from the BREP structure [5]: each node of the MSG represents a face of the BREP solid. The edge between two nodes of the graph corresponds to adjacency of two faces. This graph forms a synthetic surface-volume abstraction of the model boundary and can be used in the process of boundary recovery or domain decomposition for meshing (Fig.3). Along with the topological connectivity between faces any extra information like average distance to neighboring faces, faces area, and other CAD model properties can be stored at the graph nodes, linking the MSG with the other known graph formalism of CAD model description – the so called Multiresolutional Reeb Graph (MRG) [5]. This graph is defined by obtaining a suitable function over the 3D object boundary (i.e. geodesic curvature or distance to neighboring faces). Then the function value range over the object is split into a number of sub ranges. A region of the solid object corresponds to each sub range, and each region forms the node of the MRG. Edges of the MRG are obtained from the BREP model connectivity of shells (topological sub-volumes) [6]. The MRG can be stored as additional information at MSG graph nodes.

We use Delaunay 3D mesh generation [8] and boundary recovery is based on the known node insertion technique [2]. In a standard implementation model faces are recovered using the ad-hock order obtained directly from the BREP model without reference to meshing. However, analyses of the Signature graph provide extra data on the influence of internal geometrical constraints on boundary recovery. Here the boundary is reconstructed selectively, using a special order of faces to be recovered. A minimum spanning tree of the MSG is constructed [9] and edges of the tree provide the order of face reconstruction during boundary recovery. Due to the properties of the graph [5] and its tree [9], in problematic geometrically over-constrained region boundaries are recovered in an optimal way, resulting in insertion of fewer nodes. To summarize, in our proposed approach the following steps are performed:

- CAD repair of the BREP model to ensure model conformity
- Generation of the MSG with attributes for the BREP model
- Construction of the MSG minimal spanning tree
- Surface mesh generation on the external and internal boundary faces
- Delaunay volumetric mesh generation
- Boundary recovery for the MSG tree faces using the node insertion technique
- Boundary recovery for the remaining faces, not belonging to the tree.

3. Results and Discussion

The method has two distinct advantages. Firstly, it provides a generic graph method for description of the BREP geometrical complexity. Secondly, it formulates an algorithm for optimal boundary recovery of the geometry with multiply connected domains. The algorithm also ensures significant reduction in the number of nodes (around 43 per cent), inserted in the vicinity of the internal and external boundaries, as shown in **Table 1.**

Table1. Number of nodes in traditional and proposed boundary recovery.

Model	Domains	Traditional	Proposed	Reduction, %
Partitioned Cube	734	9327	6249	49.26
Fault	57	256349	182008	40.84
Fractured stone	289	1178252	853223	38.94

The cube geometry is a synthetic test case with internal faces corresponding to quadratic curvilinear patches. For details we are referring the reader to a full version of the paper to appear next year. Interestingly, the reduction in the number of nodes as expected improves the speed of the numerical flux calculation for the numerical solvers of hyperbolic transport problems [3]. Using our new meshing approach for flux computations, based on the Schlumberger analytical pressure solver (GREAT) for the diffusion equation (Darcy flow) we managed to obtain nearly 50% speed increase in flux computation for multiphase simulations due to the reduced number of nodes.

4. Conclusion

We have developed a new graph approach for the BREP model analyses, ensuring significant reduction of the number of nodes during selective boundary recovery, based on the properties of the minimal spanning tree of the MSG graph.

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