HEXAHEDRAL SHEET EXTRACTION

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ABSTRACT

This work presents an algorithm for coarsening all-hexahedral meshes while maintaining all-hexahedral connectivity. The dual of a mesh is described and the constraints on hexahedral mesh modification are explained. This paper then describes how mesh coarsening can be performed by using the information contained in the dual.

Keywords: mesh generation, hexahedral meshing, mesh coarsening

1. INTRODUCTION

Three-dimensional finite element analysis (FEA) is an important design tool for scientists and engineers. Before the analysis can begin, a mesh must be created for the model. There is currently significant research being devoted to the generation of such meshes. Tetrahedral meshes are well developed and have been incorporated in numerous software packages. Hexahedral meshes provide some advantages over tetrahedral meshes but are currently more restrictive in the geometrical shapes they can fill[1][2].

The current method of choice for meshing three-dimensional geometries with all-hexahedral elements is sweeping. Recent advancements have made sweeping algorithms versatile tools for solving numerous meshing problems. Sweeping algorithms can handle multiple source and target surfaces[3][4], as well as non-planar, non-parallel source and target surfaces[5]. Other algorithms have been developed with the objective of creating an arbitrary three-dimensional all-hexahedral mesh. One example is whisker weaving[6]. The whisker weaving algorithm can create meshes on complex, non-sweepable geometries. But, whisker weaving does not guarantee mesh quality and often produces initial meshes that are unsuitable for analysis due to inverted elements. Such meshes can be very difficult to untangle[7].

This paper introduces an algorithm used to modify an existing mesh by removing sheets of hexahedral elements. This algorithm can be used to coarsen the mesh or improve mesh quality by removing poor quality elements from the mesh. It can also be used to modify swept meshes so that the linking surfaces take on an unstructured configuration.

2. THE DUAL OF A MESH

An all-hexahedral mesh can be represented by its dual. Murdoch[8] discussed the possibilities of creating an all-hexahedral mesh for any arbitrary three-dimensional volume by using the properties of the dual. Two important properties of the dual that Murdoch discussed are: 1) it provides an alternative geometrical representation of a mesh and 2) it explicitly defines the global connectivity constraints of a mesh. Although the work presented in this paper does not deal with the initial mesh generation process the methods are best presented and discussed with an understanding of the dual of a mesh. For a definition of the terms used in describing the dual see [8].

Others have explored using the dual as a means of generating and modifying meshes. This work includes hexahedral mesh generation techniques described in [6] and [9]. A dual based approach for generating and modifying surfaces meshes has also been described in [10].

3. HEXAHEDRAL MESH MODIFICATION CONSTRAINT

For all-hexahedral mesh modification there are two important properties of dual chords that must be considered: 1) A chord that begins on a boundary of the meshed region must terminate on the boundary of the meshed region and 2) a chord that does not begin on the boundary must form a closed loop. These properties dictate that any insertion or extraction of elements will have a global effect on the existing mesh.

Adding or removing new elements to a conformal mesh during refinement will add or remove chords to the dual. For a three-dimensional mesh, if one new hexahedral element is added to the mesh an entire twist plane with its resulting chords will be added to the dual and removing one hexahedral element will result in removing one twist plane from the dual. This may lead to many elements being added or removed from the mesh. The chords that are added to the dual will either propagate through the mesh until they reach a boundary or they will form closed loops. This global constraint that changes to the mesh must propagate according to the dual makes generalized local modifications difficult.

4. FINDING HEXAHEDRAL SHEETS

The first step to hexahedral sheet extraction is to find all the hexahedral elements of the sheet that is being extracted. A twist plane of the dual can represent a sheet of mesh elements. A user specifies the twist plane to be refined by selecting a single mesh edge. Figure 1 shows how this is done. A mesh edge is chosen and the surface dual chord that intersects it is found (dashed lines in Figure 1 represent dual chords). The surface centroids of this chord represent end points of volume chords that are part of the twist plane that is being defined. One of these volume chords is arbitrarily chosen and the hexahedral element of the first volume centroid becomes the initial element in the sheet, h_0.

Once the initial element is found the chord segments of its dual cenetroid that lie on the twist plane can be followed to neighboring elements. If the element is on a boundary the chord will terminate at the surface centroid. If a neighboring element is found that is not already in the hex sheet it is added to the sheet and each of its neighboring elements is searched in turn. This process continues until no new elements are found. The following recursive algorithm outlines this process of finding hexahedral sheets.

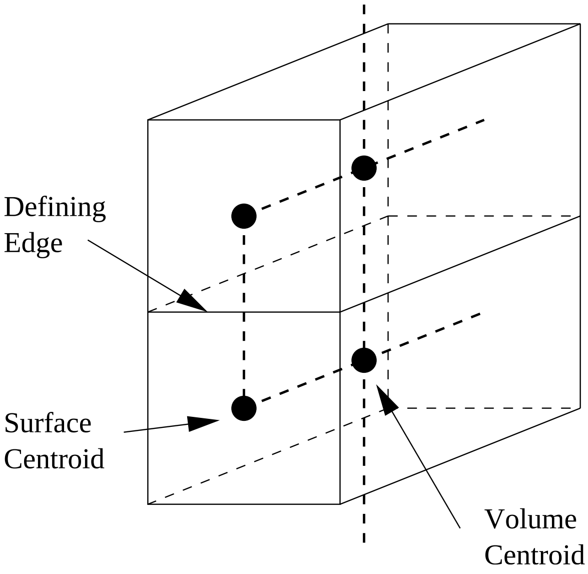

Figure 1: Defining a twist plane with a mesh edge.

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Step 1: Define the twist plane, $p$, and initial hex
        element, $h_0$, of the sheet;
Step 2: For each chord segment, $s_i$, of $h_0$ on $p$;
        Find neighboring hex element, $h_i$;
        If $h_i$ is already in sheet
          continue;
        else
          Add $h_i$ to sheet and perform Step 2 on $h_i$;
Step 3: Return hex sheet;
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5. MERGING NODES

Once the sheet of hexahedral elements that will be extracted has been found, sets of nodes that will be merged together after the extraction process are created. Each merge set contains nodes that share an edge bisected by the twist plane that defines the sheet. This grouping is cumulative so that the sets may contain more than two nodes. These larger sets will occur at self-intersections in the twist plane. Figure 2 illustrates how nodes are grouped into sets using a two-dimensional example. In this case, nodes 1 and 2 share an edge bisected by the dual. These two nodes will be grouped into one set. Nodes 3 and 4 also share a bisected edge but so do nodes 4 and 5 and nodes 5 and 6. Because the grouping is cumulative, nodes 3, 4, 5, and 6 will be grouped into a single merge set.

Before the nodes in each set are merged a check is made to ensure that the merge is possible. The governing influence in this check is the geometric entity that owns each node. The node with the lowest dimensional owner is found and a check is made to see if the other nodes in the set can be merged into it.

If the merge is possible for all merge sets the elements of the sheet are deleted from the mesh. The nodes are then merged based on owning entity; nodes owned by higher dimensional entities are merged into nodes owned by lower dimensional entities. If the owning entities of two nodes that are being merged are the same dimension, the owners are required to be the same entity and the nodes will be merged to an average location.

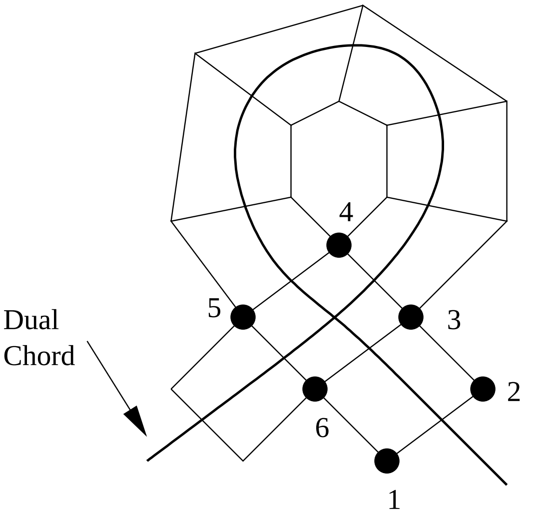


Figure 2: Grouping nodes into merge sets.

6. SHEET EXTRACTION EXAMPLES

The following figures show three examples of sheet extraction. The mesh in Figure 3a was created with the whisker weaving algorithm. Sheet extraction is used on this model to improve the mesh quality. The initial mesh has very poor quality as shown in Table 1. This mesh has sixty-one inverted elements with 6253 total elements. Figure 3b shows the mesh after sheet extraction was used to remove the inverted elements. Currently, sheets are manually selected for removal. In this example, an inverted element was arbitrarily chosen and a sheet that defined the element was removed from the mesh. By removing this sheet the element was also removed from the mesh. After a sheet was removed the mesh was smoothed and a new inverted element was selected for removal. This process continued until all inverted elements were removed from the mesh.

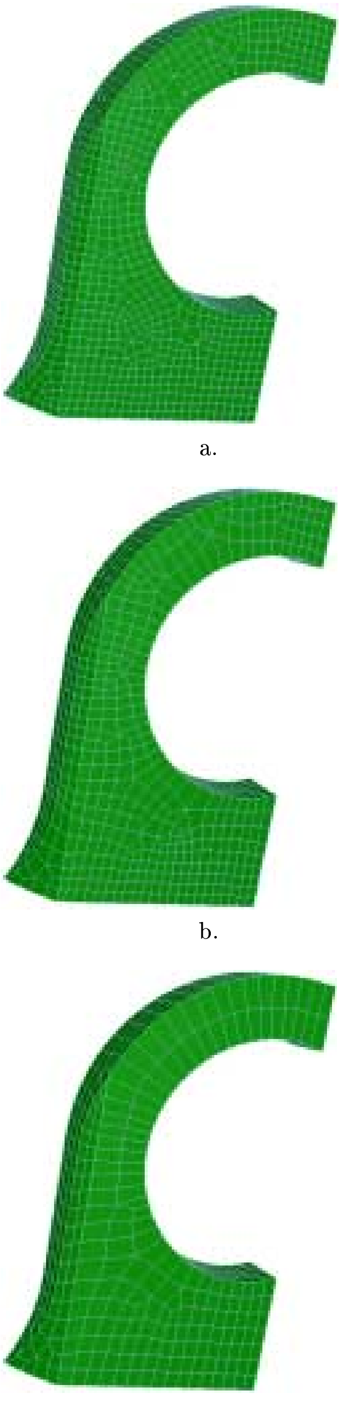
After the inverted elements were removed from the mesh sheet extraction was used to remove surface elements with six valent nodes. Again, this was done manually one sheet at a time until all six valent nodes were removed. The resulting mesh is shown in Figure 3c. The final mesh had 1612 elements with no inverted elements. Table 1 shows a comparison of the initial and final quality of this mesh.

Table 1: Quality of mesh in Figure 3 before and after sheet extraction.

	Shape		Scaled Jacobian	
	Before	After	Before	After
Average	0.6961	0.6278	0.6578	0.6569
Std. Dev.	0.1986	0.1151	0.2477	0.1735
Min.	0.0000	0.2883	-0.8900	0.2953
Max.	0.9865	0.9517	0.9963	0.9956

Figure 4a shows a mesh with a large variation in element aspect ratio. Because of the holes in the model it must be meshed by sweeping with the tapered boundary surfaces as linking surfaces. This forces the tapered surfaces to be meshed by mapping which is the cause of the variation in aspect ratio and mesh density. [11] describes a refinement algorithm that can be used to reduce this variation in mesh density by adding elements to the mesh. Similar results can be obtained with sheet extraction by extraction two partial sheets that are parallel to each other from the mesh. The first step is to determine where the mesh will transition from the coarse to fine mesh. This process is described in [11].

The next step is to remove the partial sheets and create the transition. A two-dimensional example of this process is shown in figure 5. Figure 5a shows the node where the transition will take place and an arrow indicating the direction of extraction for this example (the dashed lines show the dual chords of the mesh and the solid lines show the mesh edges). In Figure 5b the four dual chords are cut at the transition point. Next, the two middle chords are removed from the mesh in the direction of the extraction and the two outer chords are moved to an average location as shown in Figure 5c. The chords are now ready to be tied back together. The two chords in the extraction direction are tied together with the two middle chords of the unmodified mesh as shown in Figure 5d. The two outer chords of the unmodified mesh are then tied together as shown in Figure 5e. Figure 5f shows the final transition dual and mesh.



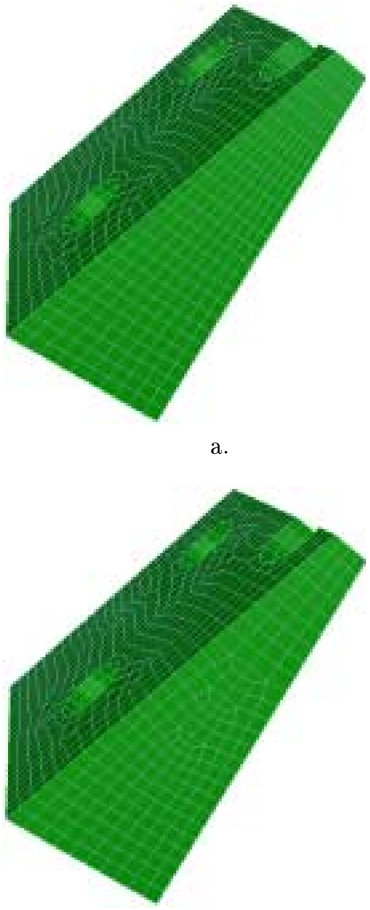
a.

b.

c.

Figure 3: Sheet extraction used to improve quality of mesh created by whisker weaveing.

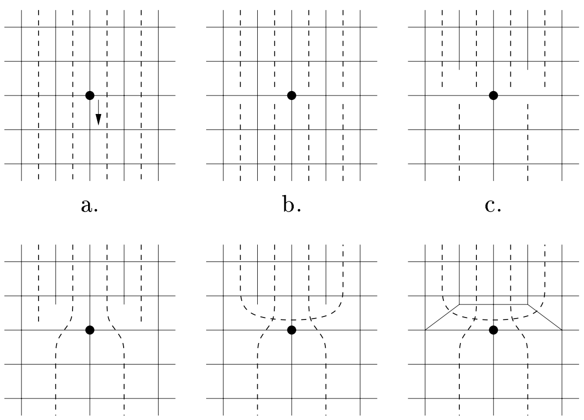
In the example in Figure 4, three successive extraction operations were performed to get the mesh shown in Figure 4b. Table 2 shows a comparison between the shape and aspect ratio quality metrics for this mesh. Notice that not only is the aspect ratio more uniform but the shape metric is also improved slightly by sheet extraction.



a.

b.

Figure 4: Example transition path sheet extraction.



a. b. c.

d. e. f.

Figure 5: Extracting elements at a transition (dashed lines show the dual).

Table 2: Quality of mesh in Figure 4 before and after sheet extraction.

	Shape		Aspect Ratio	
	Before	After	Before	After
Average	0.8511	0.8677	1.845	1.404
Std. Dev.	0.1423	0.1131	0.8637	0.3916
Min.	0.4312	0.4657	1.004	1.003
Max.	0.9989	0.9982	5.524	3.744

The final example shown in Figure 6 shows an extreme example of coarsening a mesh with sheet extraction. The original mesh shown in Figure 6a was created using the whisker weaving algorithm. Sheets were extracted from the mesh to get the final mesh shown in Figure 6c. Figure 6b shows an intermediate step between the initial and final mesh. Again, in this example sheets were selected manually by inspection until the desired coarseness was obtained.

7. CONCLUSION

This paper has presented a method to coarsen three-dimensional all hexahedral swept meshes. The method is very useful in allowing the modification of a traditionally created mesh. In its current state it can be easily incorporated into existing three-dimensional all hexahedral meshing schemes. This method can be further improved by developing routines for automatically selecting sheets for extraction during mesh improvement.

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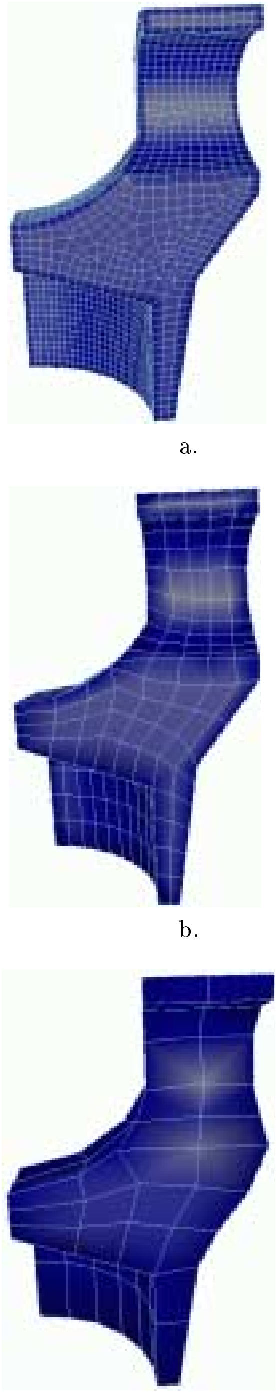

a.

b.

c.

Figure 6: Coarsening a mesh with sheet extraction.

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