Boundary Layer Mesh Generation Using Vector Fields Computed by the Boundary Element

Zhouchang Xiao, Jianjun Chen*, Yao Zheng, Jifa Zhang
Center for Engineering & Scientific Computation, Zhejiang University, China
*Corresponding author: chenjj@zju.edu.cn

Basic idea
Denote the problem domain by \( \Omega \), and its boundary by \( \partial \Omega \). A vector field over \( \Omega \) is governed by:

\[
\begin{align*}
\Delta u & = \mathbf{f} \quad \text{in } \Omega, \\
\mathbf{n} \cdot \nabla u & = g \quad \text{on } \partial \Omega.
\end{align*}
\]

Here, \( \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \) is the Laplacian operator. \( \Omega_0 \) and \( \Omega \) are the parts of boundaries where Dirichlet and Neumann boundary conditions are applied, respectively, \( \mathbf{n} \) is the unit outer normal vector at point \( p \) belonging to \( \partial \Omega_0 \), and \( u(p) \) and \( p_0(p) \) are the distributions of initial values at points belonging to \( \partial \Omega_0 \) and \( \partial \Omega \), respectively.

A 2D example indicating the capability of our method to avoid global self-intersections by analyzing the field of \( |\mathbf{u}| \). (a) The field of \( |\mathbf{u}| \). Where the interface collides, the value of \( |\mathbf{u}| \) approaches to zero. (b) The boundary layer mesh. (c) The overall hybrid mesh.

BL mesh generation
Given a surface triangulation of the domain boundary, the following three steps output a prismatic hybrid mesh:

**Step 1. Solution of the Laplacian equation**
Apply appropriate boundary conditions on different parts of \( \Omega \) according to the boundary conditions used for viscous flow computations (see Equation 1), and then compute Equation 1 by employing the BEM. Instead of creating a new boundary discretization for the BEM, we use the input surface triangulation for the BEM solution process to avoid additional consumptions of computing resources. As a result, a vector \( \mathbf{u} \) and the flux of this vector \( \mathbf{n} \cdot \mathbf{u} \) could be obtained for each element of the input surface triangulation.

**Step 2. Boundary layer mesh generation**
This step needs three user parameters, indicating the height of the first layer (denoted by \( h_0 \) hereafter), the expansion ratio of neighboring layers (denoted by \( \beta \) hereafter) and the allowed maximal number of layers (denoted by \( \eta \) hereafter), respectively. According to these parameters, we could compute the marching distance at each front node. Meanwhile, the marching direction at each front node could be computed by the boundary integration equation (i.e., Equation 2). Once the marching directions and marching distances are determined at all front nodes, a layer of prismatic elements could then be created by connecting front nodes and their duals after front propagation. Repeating this front propagation procedure for at most \( \eta \) times, we could then create semi-structured prismatic elements in the vicinity of viscous walls.

Unstructured mesh generation
If a symmetry plane is defined on the domain boundary, layered quadrilateral elements should have been created in the vicinity of common curves of the symmetry plane and viscous walls after step 2. Therefore, the surface mesh of the symmetry plane, which are initially composed of triangular elements only, need be update to accommodate these quadrilateral elements. After that, we could collect the surface triangles depicting the remaining unmeshed volume region. These triangles include those located at boundaries with non-viscous wall types and those depicting the outmost boundary of boundary layer elements. With these surface triangles as the input, we finally employ a DT based mesher to fill unstructured tetrahedra in the domain enclosed by the input surface triangles. The employed DT mesher features itself with its capability to create a boundary constrained tetrahedral mesh robustly. This feature is a key for the success of this step, where a point-to-point conformity is required between unstructured tetrahedra and boundary layer elements.

A 2D example indicating the capability of our method to avoid global self-intersections by analyzing the field of \( |\mathbf{u}| \). (a) The field of \( |\mathbf{u}| \). Where the interface collides, the value of \( |\mathbf{u}| \) approaches to zero. (b) The boundary layer mesh. (c) The overall hybrid mesh.

Meshes of the F6 model: (a) A mesh generated by the proposed method. (b) A mesh generated by a commercial tool. (c) presents a quality comparison of mesh a and mesh b.

The rotation region
The hybrid mesh used for simulating the flow structure induced by the rotation of fan blades. (a) Geometry definition. (b) A cut view of the hybrid volume mesh, which contains a structured part near boundaries and an unstructured part filling in the remaining region. (c) is a close-up view of the mesh shown in (b), where a few boundary layer elements are highlighted. (d) Viscous flow simulation results (RNG k-ε turbulence model; the fan blades rotate at a speed of 3000 rpm): Velocity vector field (back view).