Analysis of Non-Meshable Automatically Generated Frame Fields

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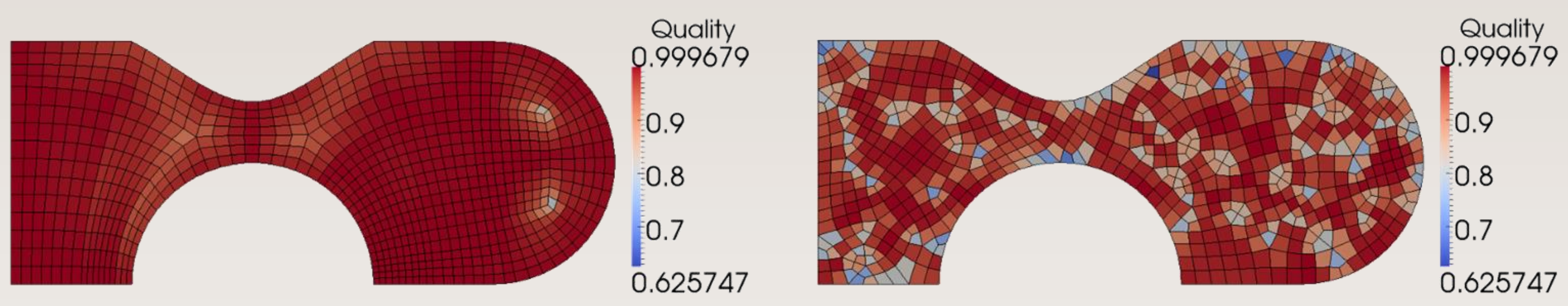
Abstract

Recent methods for frame field generation in two and three dimensions are reviewed. Frame fields generated automatically in 2D and 3D are analyzed with respect to quad and hex mesh generation. Problems are identified with automatically generated frame fields that prevent mesh generation via current methods. Specifically, there exist geometries that contain limit cycles and cannot be parameterized or decomposed by separatrices of the frame field. In 3D, singularity lines occur that minimize the field curvature but do not align with the frame field. These types of singularities make it impossible to create a mesh that both follows the frame field, and simultaneously respects the singularity as an irregular node in the mesh. Specific examples are presented that illustrate these problems. For each example, streamlines are used to help visualize properties of the frame fields, problems are analyzed, and options to potentially mitigate such problems are discussed.

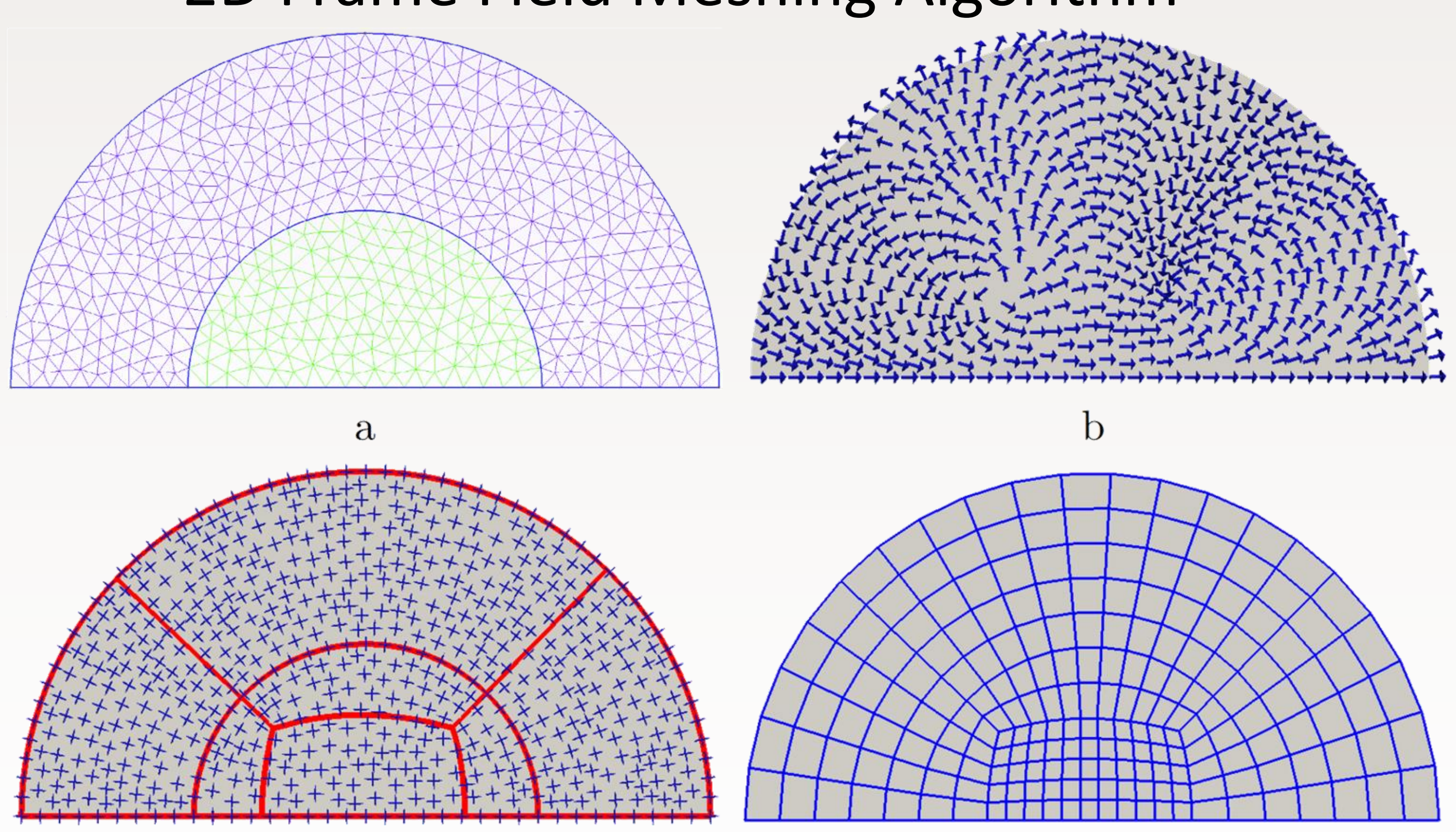
Frame Fields Automatically Generate Good Meshes in 2D



Kowalski et al. 2013

Top: A comparison between element quality (scaled Jacobian) between a frame field generated mesh (a) and on generated by an unstructured method (b).

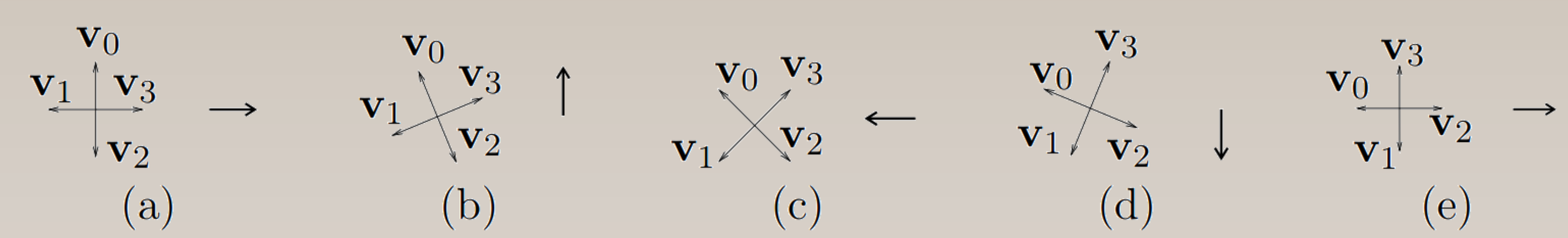
2D Frame Field Meshing Algorithm



Kowalski et al. 2013

The Geometry is Tri-Meshed (a). A frame (representation vector) is defined on each point of the boundary and Laplace's equation is solved to propagate information onto the interior (b)

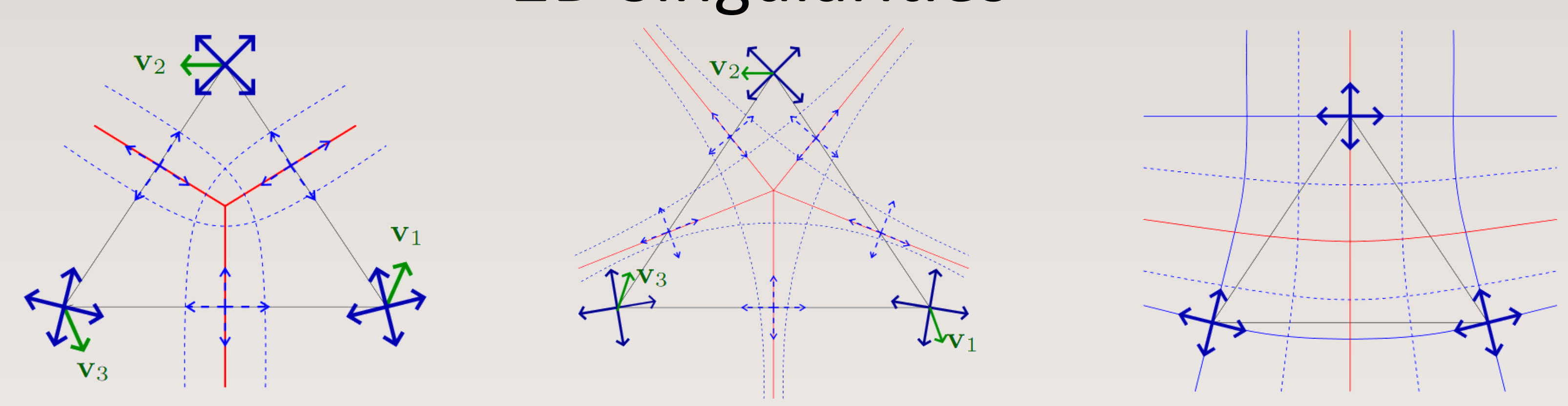
Frames and Representation Vectors



Kowalski et al. 2013

A Frame field defines a frame at each point of the geometry. Each Frame can be represented by a unique vector as shown above.

2D Singularities



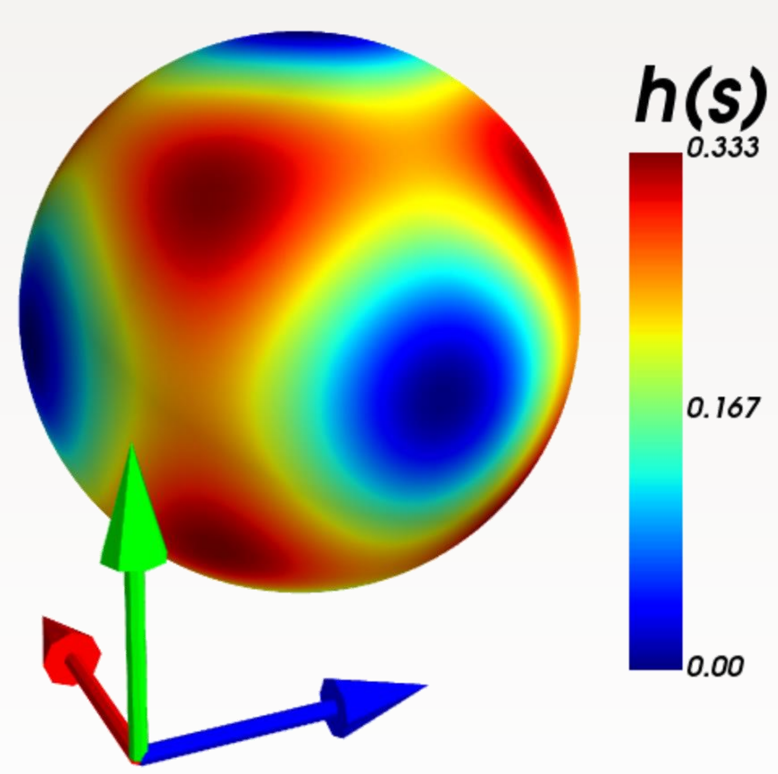
Singular Triangles

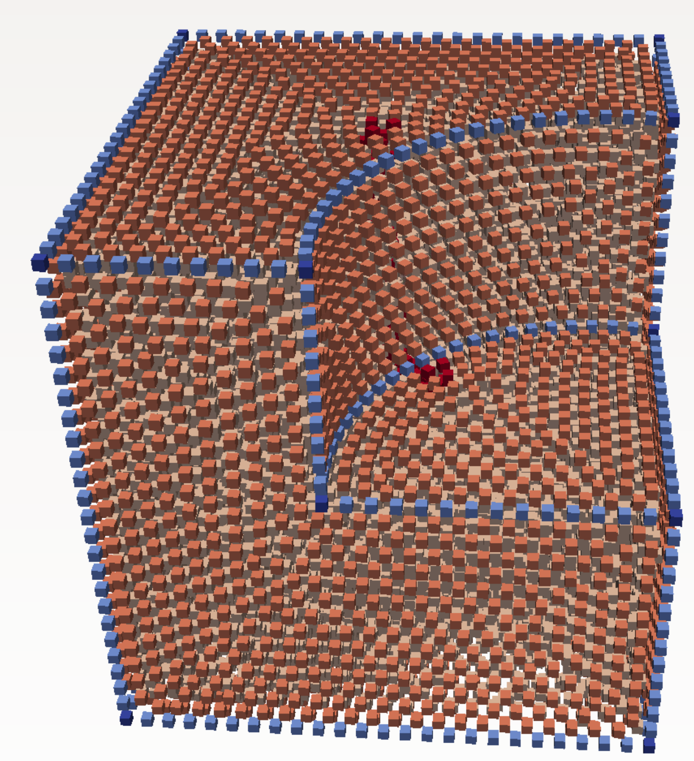
Non-singular Triangle

Kowalski et al. 2013

Singularities in a discrete 2D frame field correspond to triangular faces where a frame makes a full rotation when traversing around the triangle. Frames between nodes are determined by linear interpolation. A counterclockwise rotation corresponds to a three valent singularity (left). A clockwise rotation corresponds to a 5 valent singularity (middle)

Frame Fields in 3D





$$h(s) = s_x^2 s_y^2 + s_x^2 s_z^2 + s_y^2 s_z^2$$

In 3D, frame are represented by functions, and the change in orientation between two frames is given by the L2 distance between their representative functions

Automatic Hex Meshing with Frame Fields

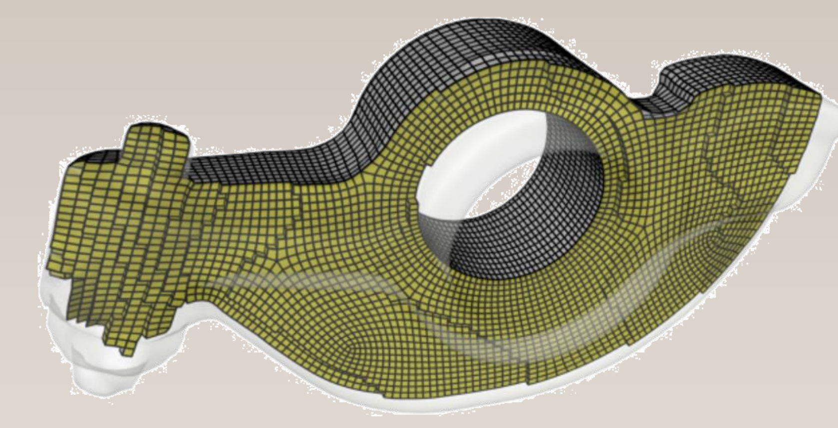
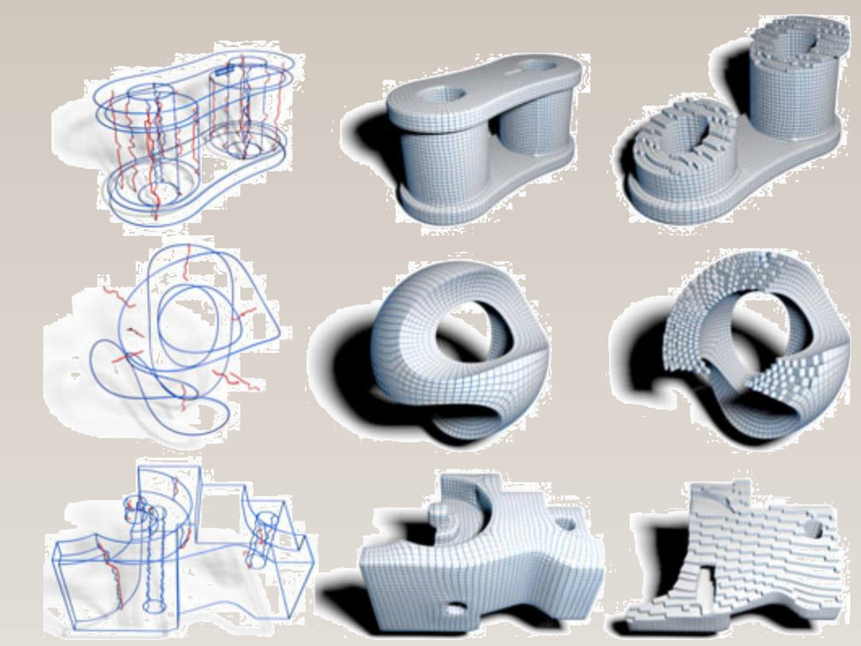
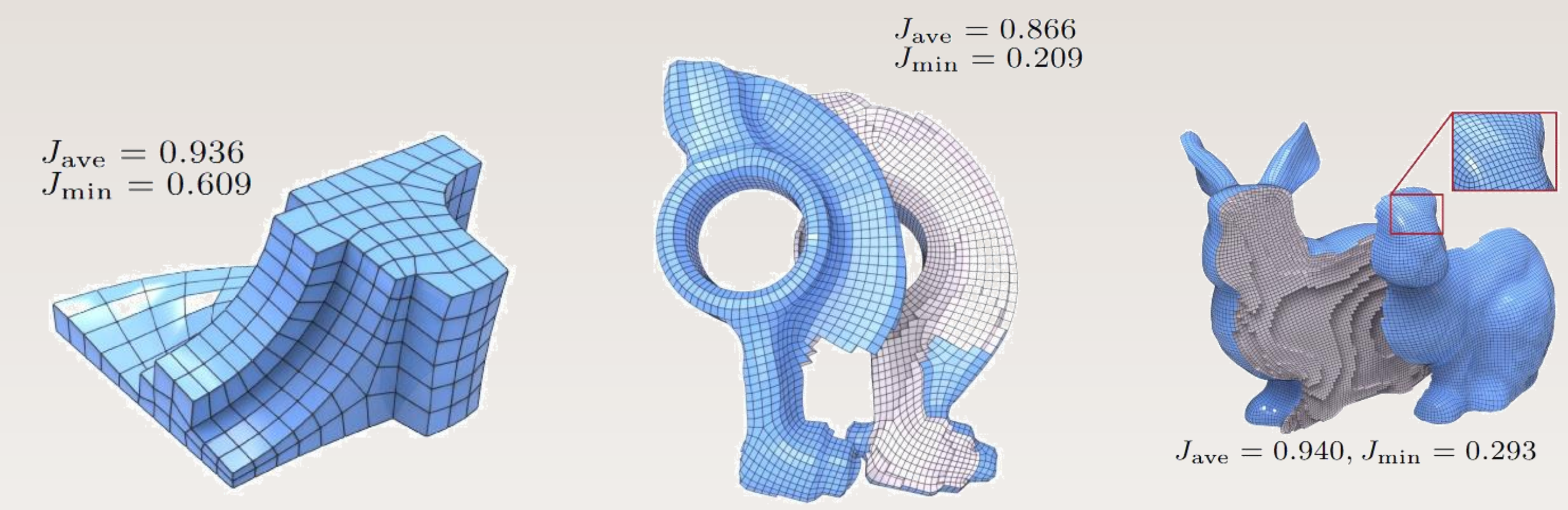


Figure 1: *Section through the rockerarm volume. Hexahedral mesh computed with* CUBECOVER.

Neiser et al. 2011



Jiang et al. 2014

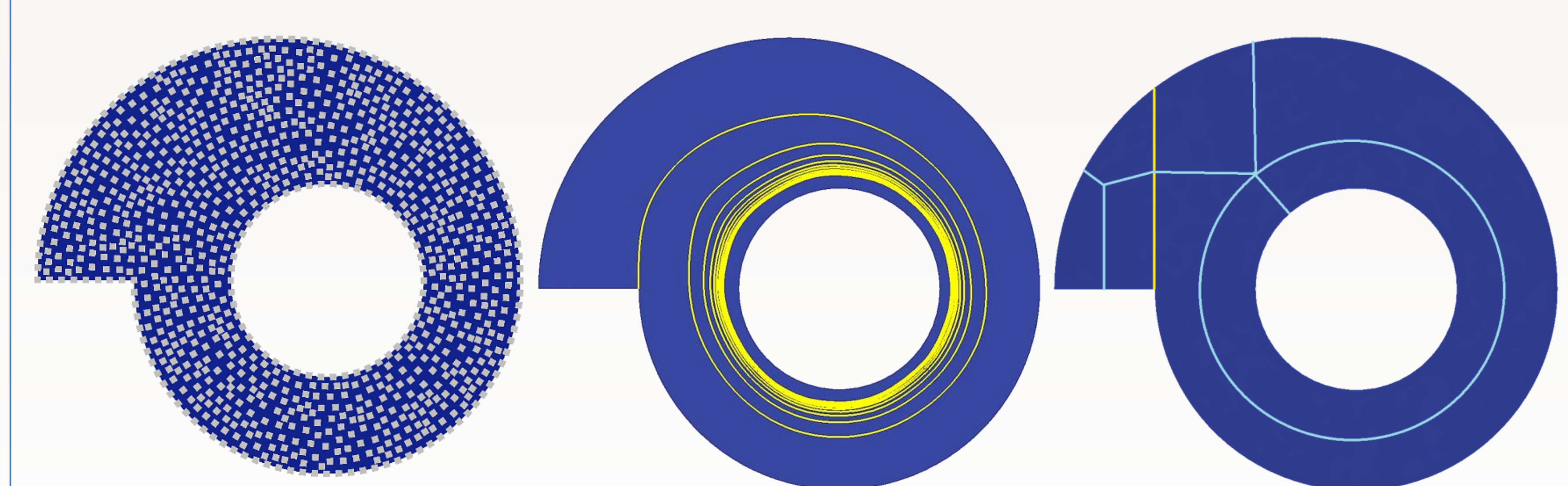


Li et al. 2012

State-of-the-art methods that produce all hex meshes from frame fields. Though these methods show promise, they can fail on even very simple geometries.

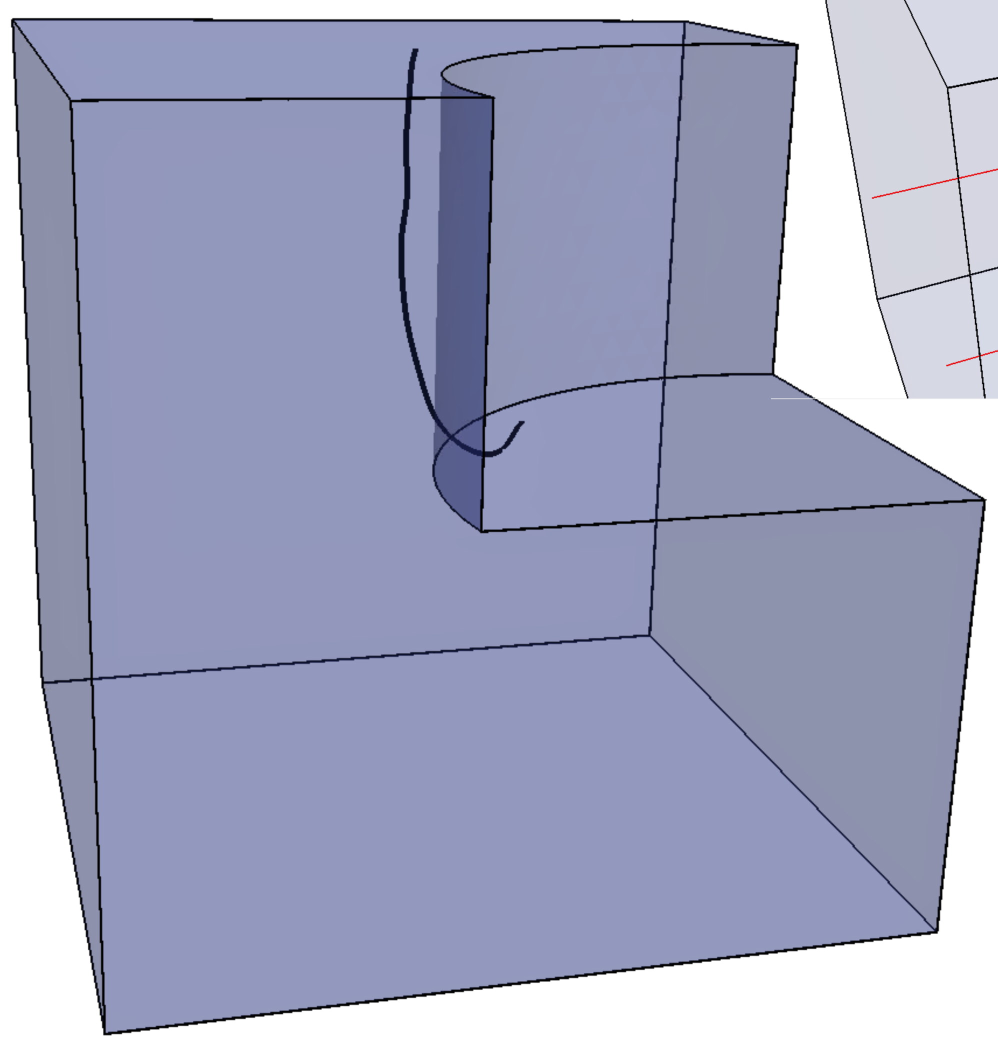
Problematic Examples

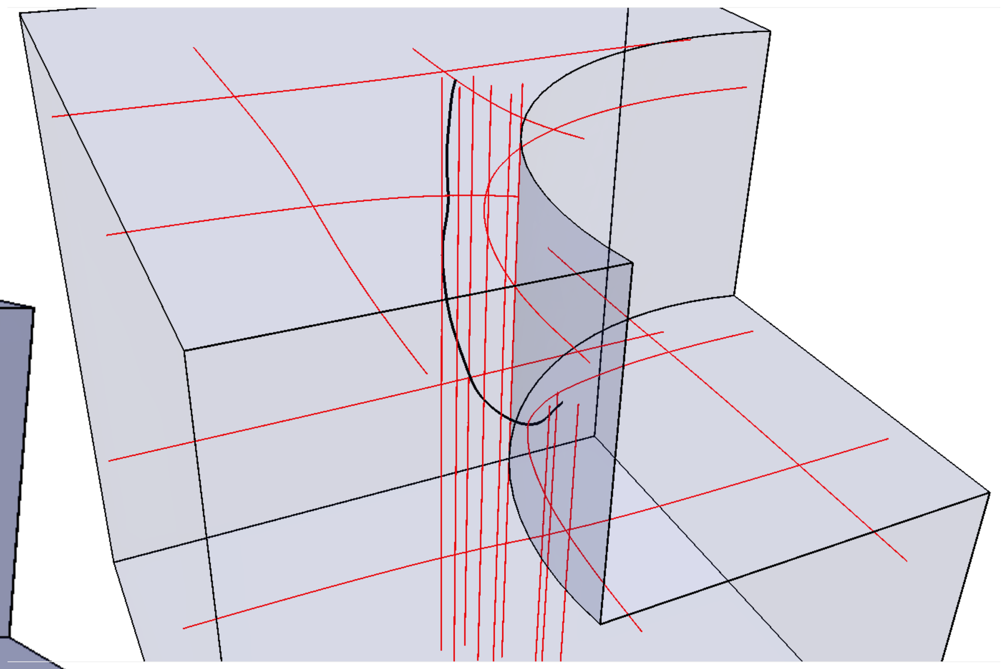
The Nautilus – Limit Cycles

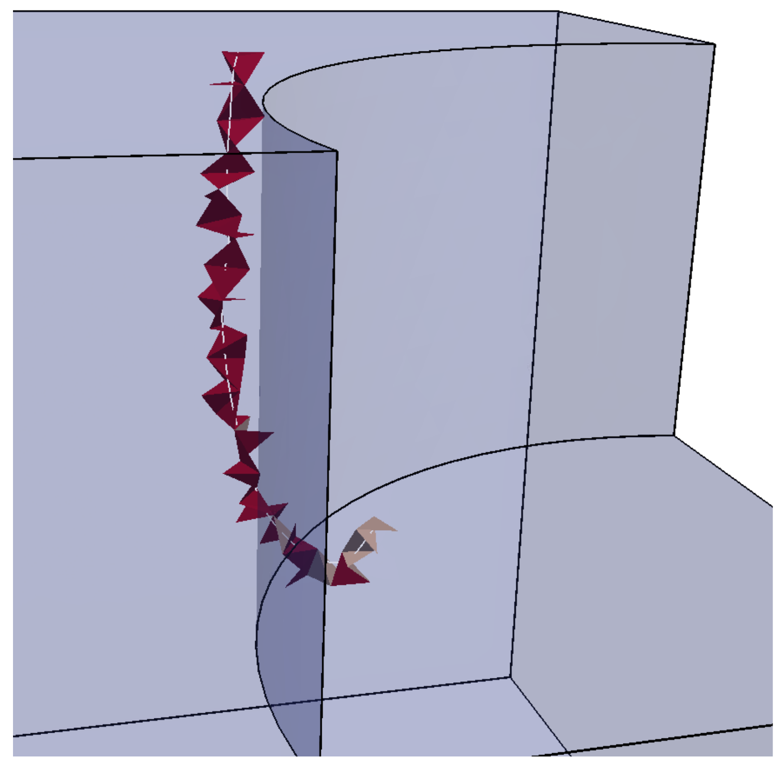


The nautilus frame field (left) contains limit cycles (yellow spiral, middle) that prevent a proper parameterization or decomposition of the geometry. If an additional interior constraint is introduced (yellow line, right), the resulting frame field decomposes the geometry into 4 sided regions

The Notch – Singularities not "Parallel" to the Frame Field



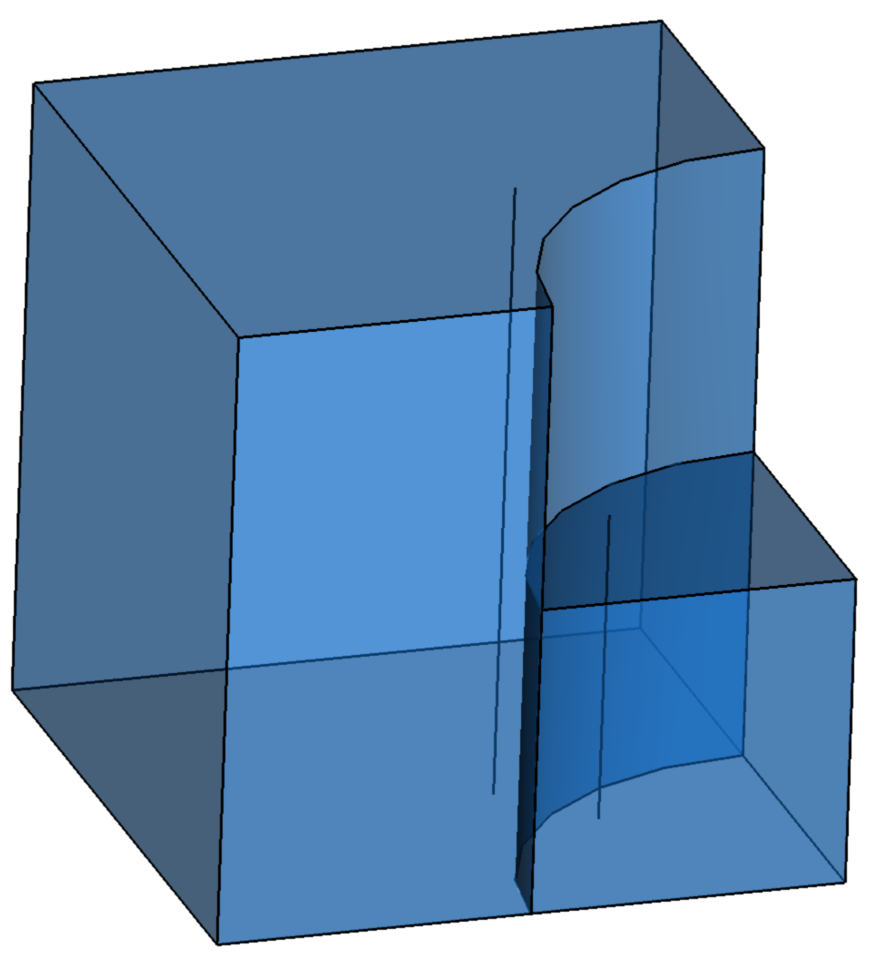


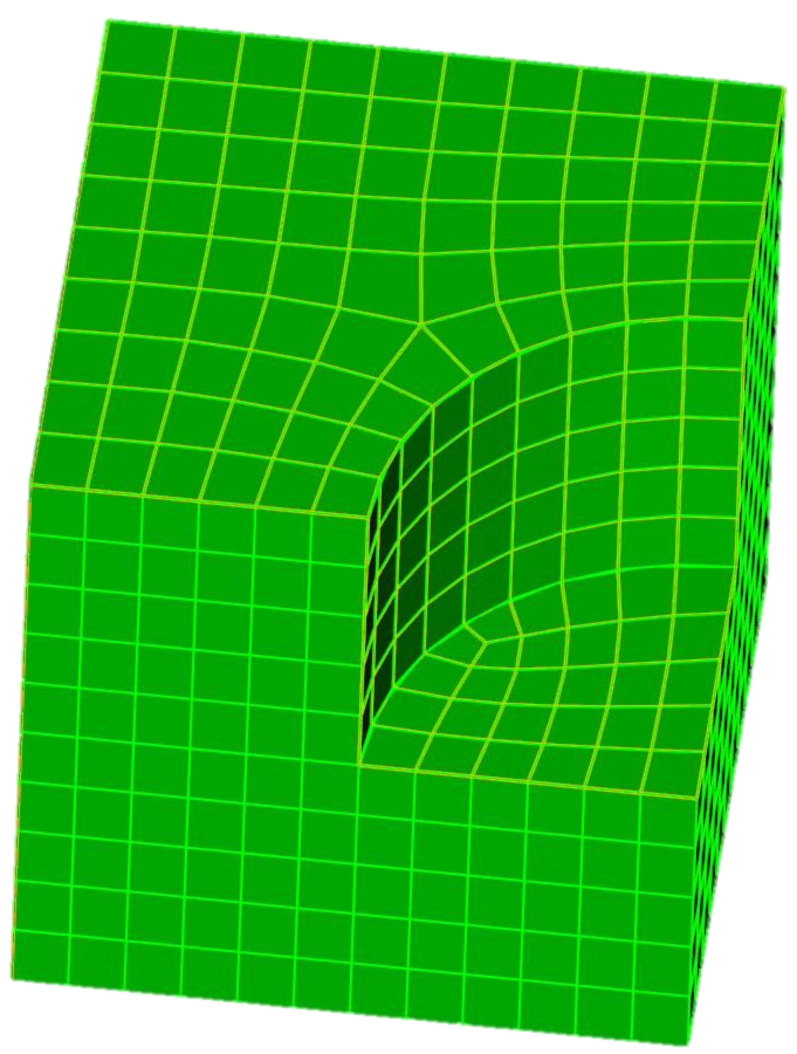


The notch geometry (left) results in a problematic Frame field because the geometric constraint (curved surface) that causes the singularities on the top surfaces to occur ends at a concave curve. This results in a singularity which is not parallel to the surrounding frame field (top right). It is impossible to associate this curve to either a three (tan faces) or five valent (red faces) singularity in a hex mesh (bottom right).

Ideas on how to fix it:

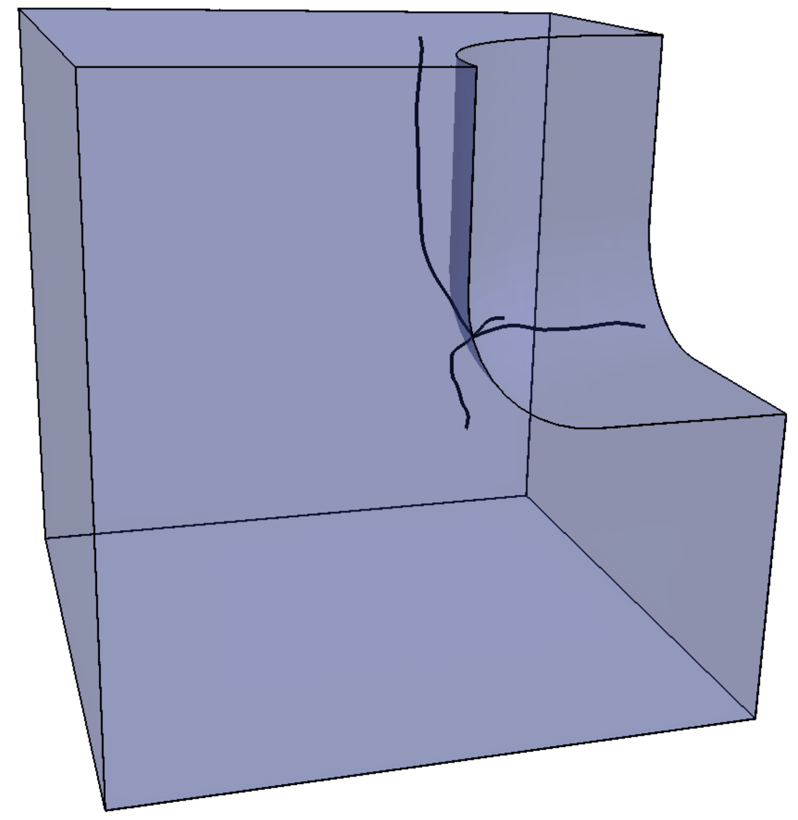
Propagate the Concavity

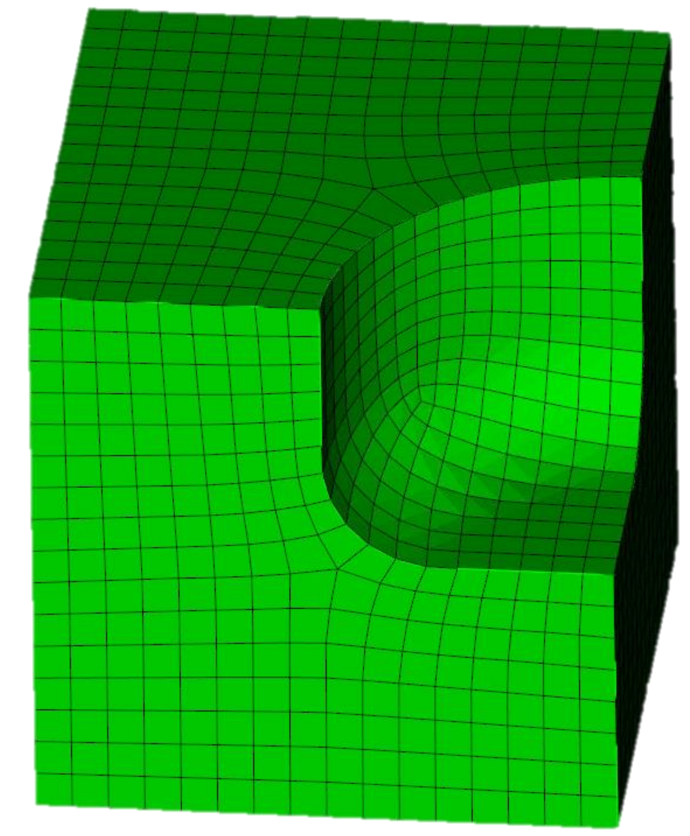




If we propagate the constraint through the geometry we get a valid singularity graph (left). This frame field corresponds to an all hex mesh (right).

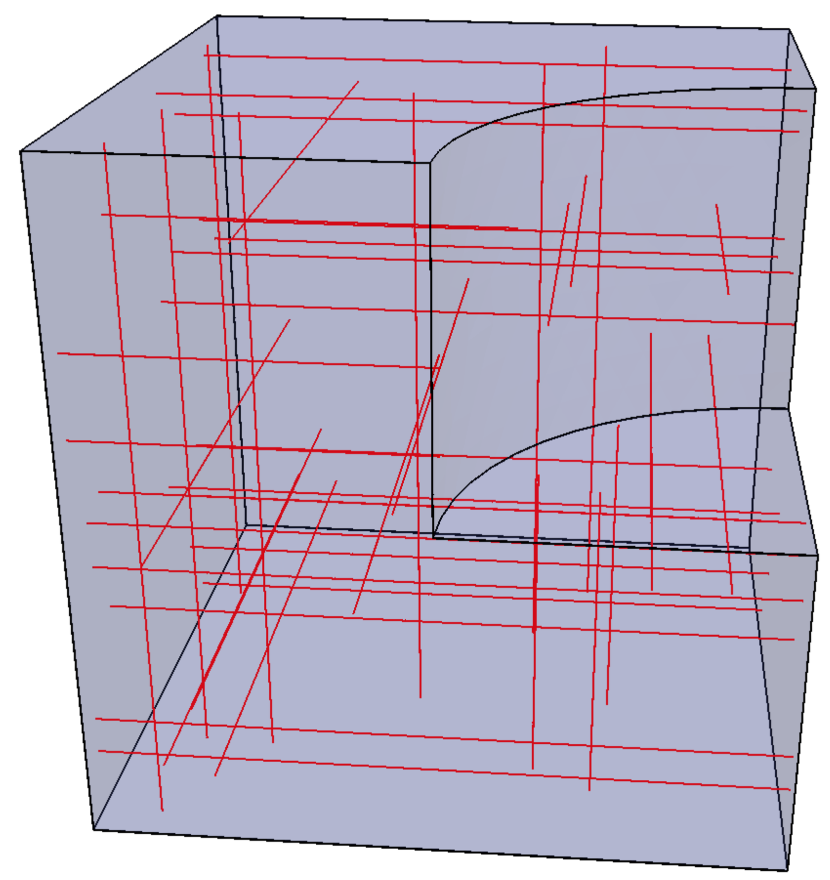
Blend the Concavity

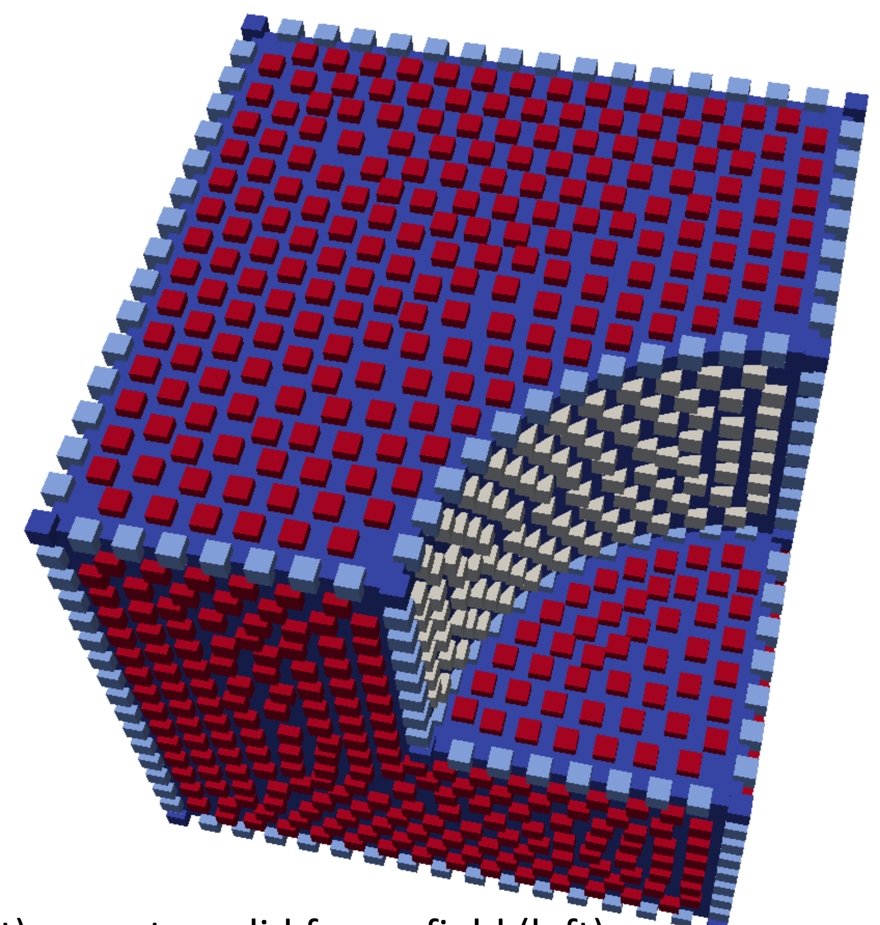




If we blend the problematic concavity we also get a singularity graph (left) that corresponds to an all hex mesh (right). Note that if design restrictions prohibit a blend in the geometry, we can achieve the same mesh topology by simply treating the curve as a side.

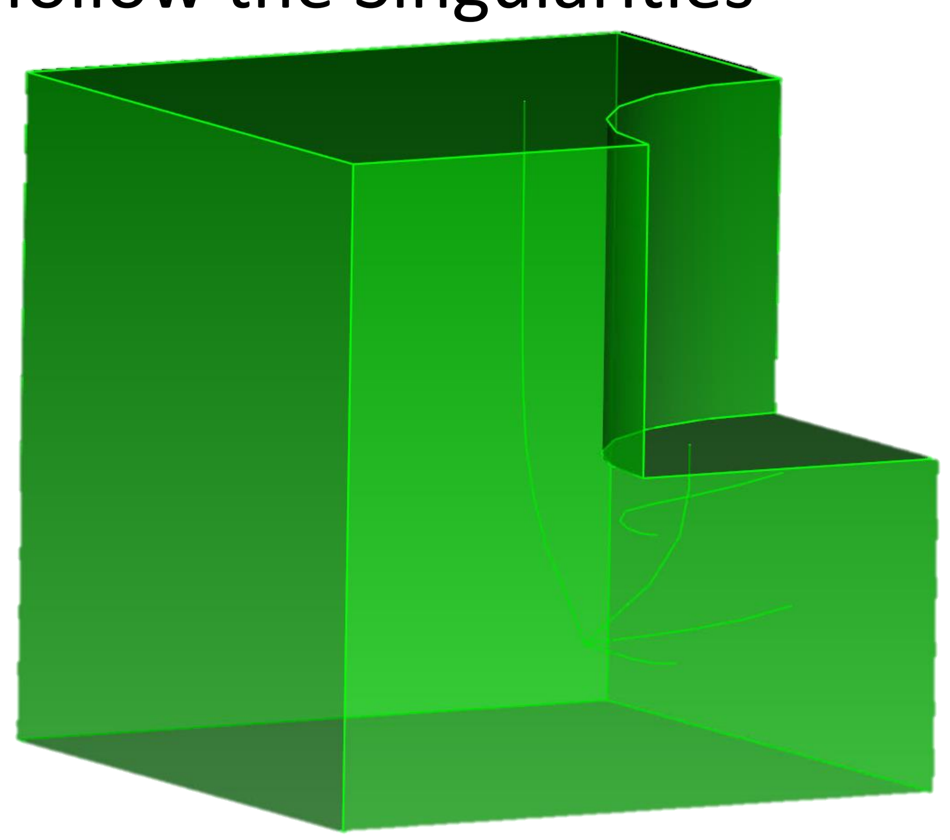
Don't Conform to Concave Face

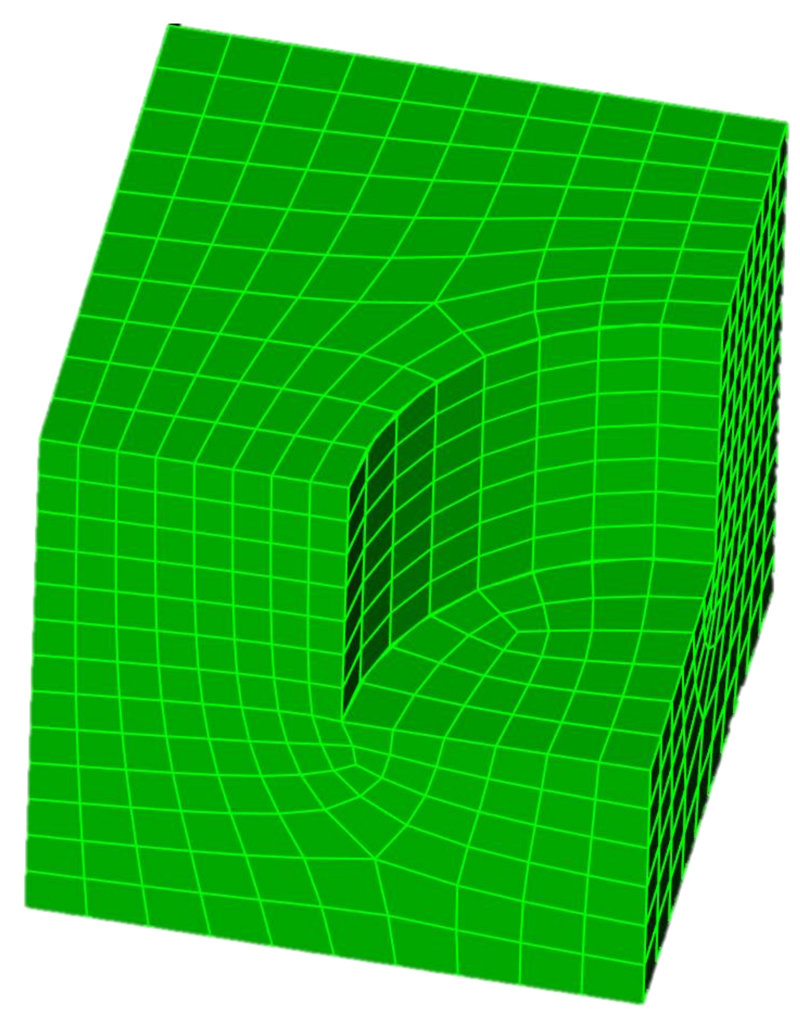




If frames don't conform to the boundary (right) we get a valid frame field (left) at the cost of lower quality elements along the boundary (mesh not show).

Constrain the Frames to follow the Singularities





A manually built hex mesh showing what might result if frames ran parallel to the singularity graph of the notch geometry. Notice the additional singularities.

Objective Function

$$\min \underbrace{\sum_{i,j\in T^1_\Omega} ||a_i - a_j||^2}_{\text{change in frame orientation}} + \lambda \underbrace{\sum_{j\in J} ||a_j - R^B_V R^B_z(\theta)\tilde{r}||^2}_{\text{boundary constraint}} + \underbrace{\lambda \sum_{i\in S} ||a_i - (\gamma R^B_{V_n} + (\gamma-1) R^B_{\hat{V}}) R^B_z(\theta)\tilde{r}||^2}_{\text{singularity alignment constraint}}$$

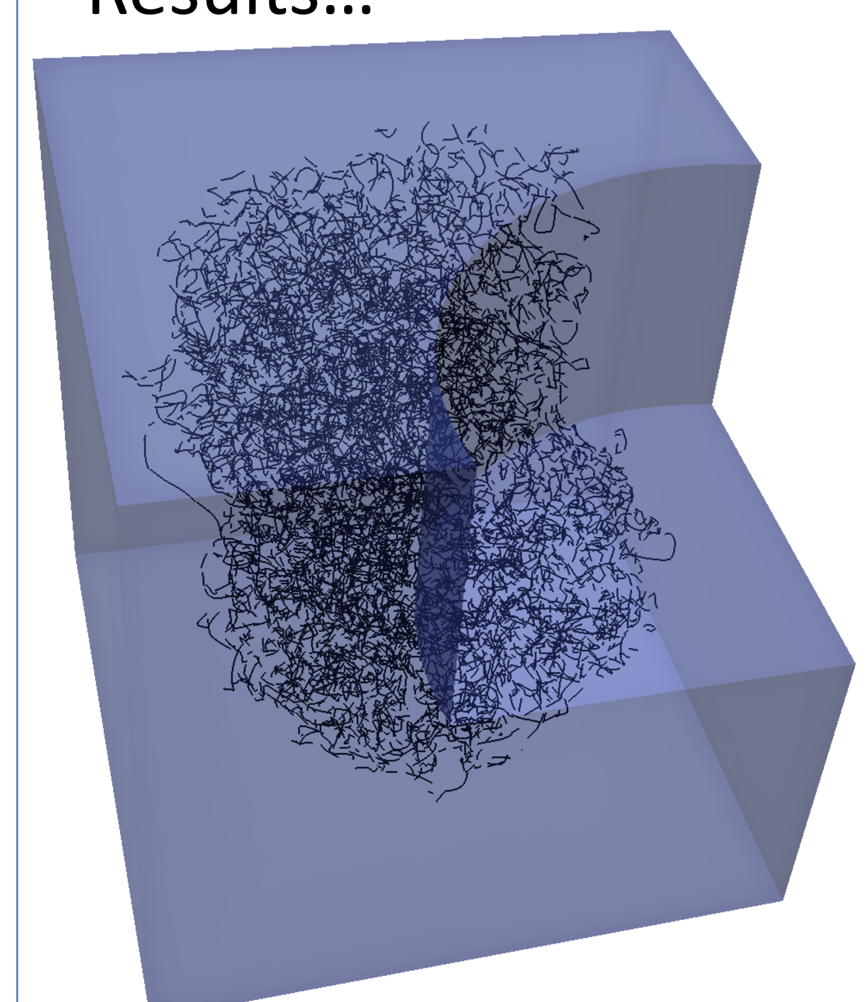
The problem of propagating boundary frames smoothly onto the interior of a 3D geometry is weakly formulated as an energy minimization problem. In an attempt to create frame fields with valid singularity graphs, we added a constraint to the objective function that penalized frames that were not parallel to a nearby singularity. For details on formulation of the objective function, see *On Smooth Frame Field Design,* Ray et al. 2015.

Algorithm

```
minimizeObjectiveFunction(); // without singularity constraint
projectOntoFeasibleSolution();
for(int i=0; i < N; i++) //gradient descent iterations
{
 findSingularities();
 addSingularityConstraintToObjectiveFunction();
 addLocalOptConstraints(); //penalizes non-feasible solutions
 minimizeObjectiveFunction();
 projectOntoFeasibleSolution();
}
```

This algorithm follows the frame field generation/smoothing algorithm from *On Smooth Frame Field Design,* Ray et al. 2015 with modification to include the singularity alignment constraint.

Results…



The proposed algorithm was unstable and led to chaotic behavior in the frame field

Future Directions

These simple examples as well as others demonstrate that current methods for 3D frame field generation do not guarantee that the frame field will correspond to an all hex mesh. However, since frame guided automatic hex meshing techniques do work on some geometries, it is important to determine necessary and sufficient for a frame field on a geometric domain to correspond to an all hex mesh. In the above examples, topological modifications to the geometric domain led to meshable frame fields. It is thus reasonable to believe that such necessary and sufficient conditions might be stated in terms of the topology of the geometric boundary. It is further important to explore a paradigm of how to satisfy the necessary and sufficient conditions by making topological decisions programmatically or by user guidance.

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