NON-INTERPOLATIVE CURVED HIGH-ORDER MESH GENERATION

How to obtain high-quality and high-accuracy meshes

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ABSTRACT

We present a novel methodology to generate curved high-order meshes featuring **optimal mesh quality and geometric accuracy**. We combine into a single objective function a distortion measure and a geometric $\mathcal{L}_2$-disparity measure. The former takes into account the mesh quality, and the latter takes into account the geometric error of the mesh. The proposed technique has several advantages. First, we are not restricted to interpolative meshes and therefore, the resulting mesh **approximates the target domain in a non-interpolative way**, further increasing the geometric accuracy. Second, we are able to generate a series of meshes that **converge to the geometry and its normals with exponential rate while obtaining high-quality elements**. Furthermore, we apply the method to **3D examples**.

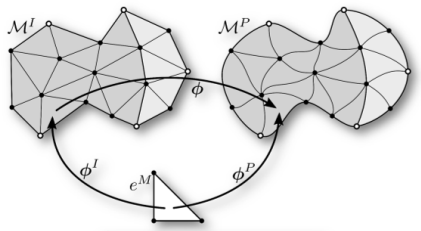
GOAL

Generate curved high-order meshes that

1. **High-quality elements**
   Low interpolation error and condition number
2. **High geometric accuracy**
   Geometric error does not pollute the numerical solution.
3. **Exponential** convergence to the target geometry

NON-INTERPOLATIVE MESH CURVING

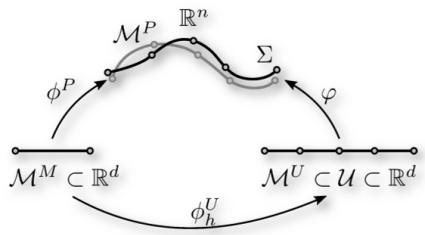
Mesh Distortion [1,2]



$$E_Q(\phi) = \|M\phi - 1\|^2$$

$$M\phi(\mathbf{y}) = \frac{|\mathbf{D}\phi(\mathbf{y})|^2}{n\sigma_\delta(\mathbf{D}\phi(\mathbf{y}))^{2/n}}$$

$\mathcal{L}_2$-disparity Measure [3]



$$E_\Sigma(\phi^P; \phi^U) = \left\|\phi^P - \varphi \circ \phi^U\right\|^2$$

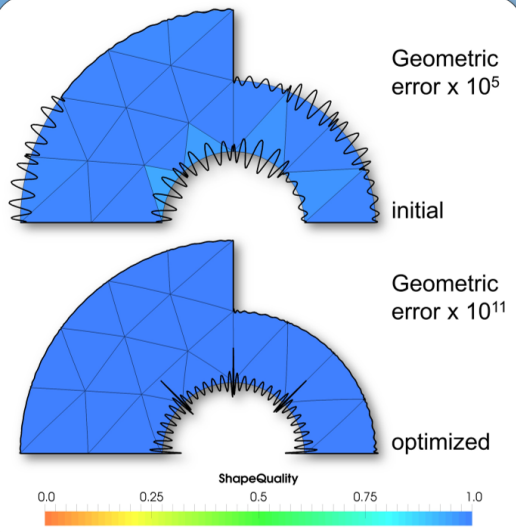
Alternative distance measure in [4]

Functional Combination

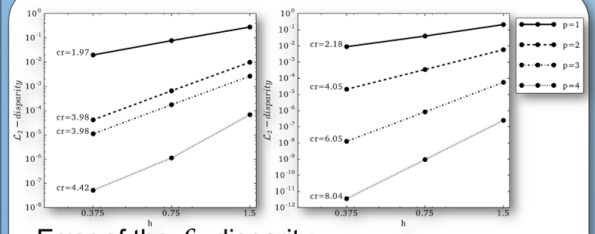
$$E_\mu(\phi^P; \phi_k^U) = E_Q(\phi) + \mu \underbrace{\sum_{k=1}^{N} E_{\Omega_k}(\phi_k^P, \phi_k^U)}_{E_\Omega = \left(d(\mathcal{M},\Omega)\right)^2}$$

1. **Apply penalty method**
   where μ is a penalty parameter
2. **Check $\mathcal{L}_2$-disparity measure**
   Mesh distance to a CAD model

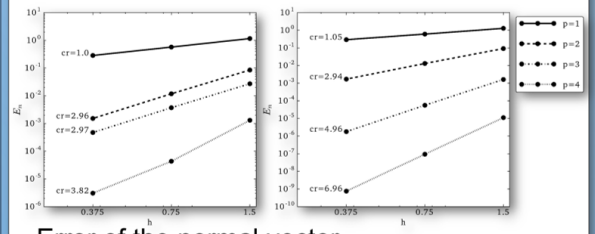
EXAMPLE - P = 10



EXPONENTIAL CONVERGENCE RATE

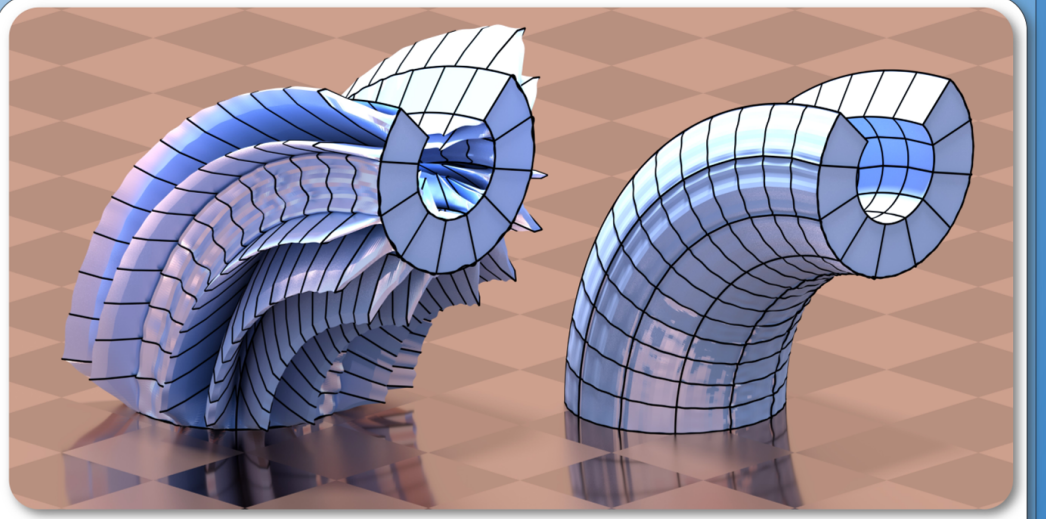


Error of the $\mathcal{L}_2$-disparity



Error of the normal vector

THREE-DIMENSIONAL MESH - P = 4



initial Geometric error x 10^5 optimized

REFS

[1] **A. Gargallo-Peiró, X. Roca, J. Peraire and J. Sarrate**. *Optimization of a Regularized Distortion Measure to Generate Curved High-Order Unstructured Tetrahedral Meshes*. Int Numer Meth Eng 103 (5), 342-363 (2015).
[2] **E. Ruiz-Gironés, X. Roca and J. Sarrate**. *High-order mesh curving by distortion minimization with boundary nodes free to slide on a 3D CAD representation*. Comput Aided Des 72, 52-64 (2016).
[3] **E. Ruiz-Gironés, J. Sarrate and X. Roca**. *Defining an $\mathcal{L}_2$-Disparity Measure to Check and Improve the Geometric Accuracy of Non-Interpolating Curved High-Order Meshes*. 25th International Meshing Roundtable, 2015.
[4] **T. Toulorge, J. Lambrechts and JF Remacle**. *Optimizing the Geometric Accuracy of Curvilinear Meshes*. J Comput Phys 310, 361-380 (2016).