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Symmetry-Aware 3D Volumetric Mesh Generation - An Analysis of Performance and Element Quality

Florian Rudolf[a,*], Josef Weinbub[a], Karl Rupp[a,b],
Andreas Morhammer[a], Siegfried Selberherr[a]

[a] *Institute for Microelectronics, TU Wien, Vienna, Austria*
[b] *Institute for Analysis and Scientific Computing, TU Wien, Vienna, Austria*

Abstract

We exploit geometrical symmetries to accelerate three-dimensional volumetric mesh generation by using a simple, yet flexible, symmetry-aware mesh generation pipeline. Performance benchmarks show, that for complex geometries, especially geometries with very thin but large layers, speed-ups of 3 to 10, in some cases even up to 50 can be achieved at comparable element quality.

Keywords: symmetry-aware mesh generation; symmetry; rotational symmetry; ViennaMesh

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1. Introduction & Related Work

Symmetric objects have the powerful property, that they can be constructed from a smaller piece of the object using rigid transformation (Fig 1). We investigate how a mesh generation algorithm can take advantage of that property to boost its performance while producing meshes which preserve those symmetries, hence maintaining expected symmetrical invariants in simulations of symmetric processes. In particular, we investigate possible enhancements of the mesh generation process in the case of rotational symmetries, as this case exhibits the highest optimization potential. This work has been motivated by devices and components used in microelectronics simulations, where complex geometries, especially very thin but large layers, are common.

However, symmetry-aware volumetric mesh generation algorithms are rare and algorithms for automatically detecting symmetries are non-trivial. Simple algorithms for detecting rotational symmetries in two and three dimensions suffer from problems like requiring additional information, e.g., an axis of symmetry [1,2]. Algorithms with support for detecting non-exact symmetries [3,4] are more practicable. Symmetry-aware surface mesh processing is a popular area of research in the field of computer graphics [5–7]. Theoretical background for symmetry-aware volumetric mesh generation using a generalized Fourier transform is available [8], but a software implementation is lacking.

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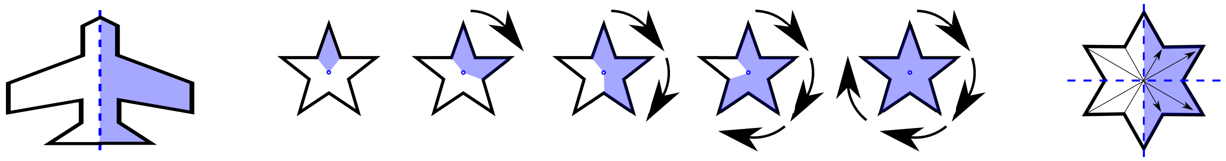
[*] Corresponding author.
E-mail address: rudolf@iue.tuwien.ac.at



(a) Reflective symmetry.

(b) Rotational symmetry.

(c) Point symmetry.

Fig. 1: Example objects with symmetries.

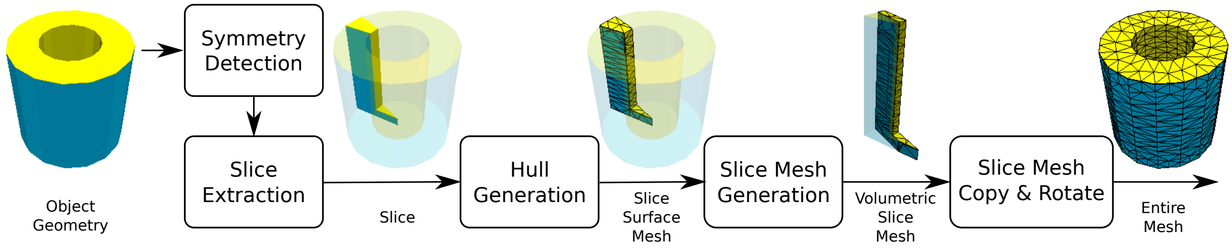


Fig. 2: Our symmetry-aware mesh generation pipeline. First, the symmetry detection algorithm identifies rotational symmetries. A slice of the object according to the rotational symmetry is extracted. Then, a surface mesh is created which is used to generate a volumetric slice mesh. Finally, the full mesh is obtained by n rotated copies of the mesh slice.

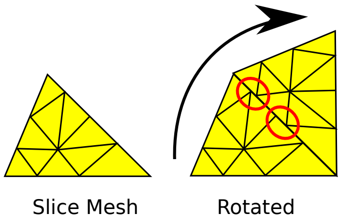


Fig. 3: Illustration of issues with non-matching interface meshes after rotation.

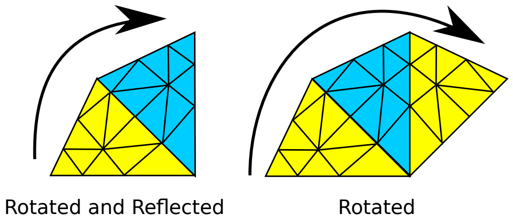
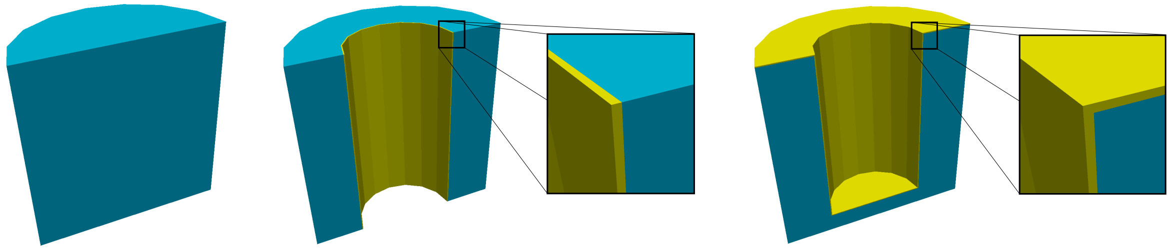


Fig. 4: Full mesh creation for objects with even rotational symmetry order and reflective symmetry.

2. Symmetry-Aware Mesh Generation

In this section we present our approach for a symmetry-aware mesh generation pipeline (Fig. 2). Although we focus on tetrahedral mesh generation in this work, our approach can also be applied to other mesh types, like hexahedral meshes. First, a symmetry detection algorithm [3] is used to identify all rotational symmetries of the object. Then, the highest rotational symmetry order n is selected and two planes with a common intersection at the rotational axis and a span angle equal to the angle of the rotational symmetry are used to extract a slice of the object. To avoid problems with interface compatibility between two neighboring slice meshes (Fig. 3), a surface mesh of the slice is generated, where all surface elements on one slice plane are equal to the rotated surface elements of the other slice plane using the rotational symmetry angle. The slice surface mesh is then used by a mesh generation algorithm to create a volumetric mesh while preserving the surface elements. Finally, the full mesh is obtained by n rotated copies of the mesh slice.

Objects with an even rotational symmetry order n and a reflective symmetry with the reflecting plane including the axis of rotational symmetry allow for additional optimization: The full mesh can be created by rotating and reflecting the slice mesh (Fig. 4). In that case no surface mesh generation is required, but the rotational order of the full mesh is halved.



(a) Object of category A, a *right regular* prism

(b) Object of category B, a *right regular* prism with a hole in its center and a thin layer coating its inside

(c) Object of category C, a TSV-like object with a thin layer (object of category C)

Fig. 5: Cuts of representative objects for each category

3. Results

To evaluate the performance and the element quality of our approach, benchmarks have been performed utilizing the free open source meshing framework ViennaMesh[1] [9], which in turn uses Tetgen [10] for three-dimensional (3D) volumetric tetrahedral mesh generation. The benchmarks have been carried out on an Intel Core i7-3770 workstation. The entire meshing pipeline is based on serial execution; however, future work will investigate the potential for parallelism to further increase the overall performance. The objects used for benchmarking have been motivated by through-silicon via (TSV) structures, which are components for connecting silicon dies in an integrated circuit [11]. TSVs have large but thin layers with thickness-length-ratio of about 10^{-4} to 10^{-3}. Three categories of test objects with rotational symmetries of order 4, 8, 16, 32, 64, and 128 (Fig. 5) have been utilized. Objects of category A are *right regular* prisms, objects of category B are *right regular* prisms with holes in their center and a thin layer coating their insides, and objects of category C are simple TSV-like objects, also with a thin layer. The thickness-length-ratio of the layers for objects of category B and C is 5×10^{-3}. For each benchmark the execution time of the non-symmetry-aware mesh generation is compared to the execution time of our pipeline using identical mesh generation parameters - being the maximum radius edge ratio, the minimum dihedral angle, and the element size - for both processes. For each test object and each rotational symmetry order, 27 benchmarks have been conducted using different sets of mesh generation parameters. In addition, we estimate the performance of the symmetry detection step by worst-case benchmark data from previous work [3].

A comparison of average element quality histograms over all benchmarks for the maximum radius-edge-ratio and the minimum dihedral angle for objects of category B and C is given in Fig. 6. It can be seen that the overall element quality is not significantly reduced, confirming that the performance gains due to symmetry-aware meshing are not achieved at the expense of element quality.

The performance benchmark results are presented in Fig. 7. For objects of category A, the speed-up is at most moderate, often even lower than 1. This is due to very fast volumetric mesh generation for the simple shapes of right regular prisms, causing the overhead of slice extraction and full mesh recombination to be significant. For objects of category B and C, the generation of quality mesh elements in the thin layers is time consuming. Therefore, the pipeline overhead is much less significant and speed-ups of 3 to 10, in some cases up to 50, can be observed. However, for higher rotational symmetry orders, the pipeline overhead increases due to the super-linear complexity in the slice extraction algorithm resulting in lower speed-ups. As depicted in Fig. 7d, the element count directly correlates with the speed-up for test objects of all three categories and therefore is the main indicator for speed-up in symmetry-aware mesh generation.

Test objects of category A, B, and C are synthetic. A benchmark using a practical TSV with a rotational symmetry order of 24 and 6 layers with an even smaller thickness-length-ratio of 7×10^{-4} yielded a speed-up of 22, which is in agreement with what is predicted by the test objects and thus confirms the validity of our tests.

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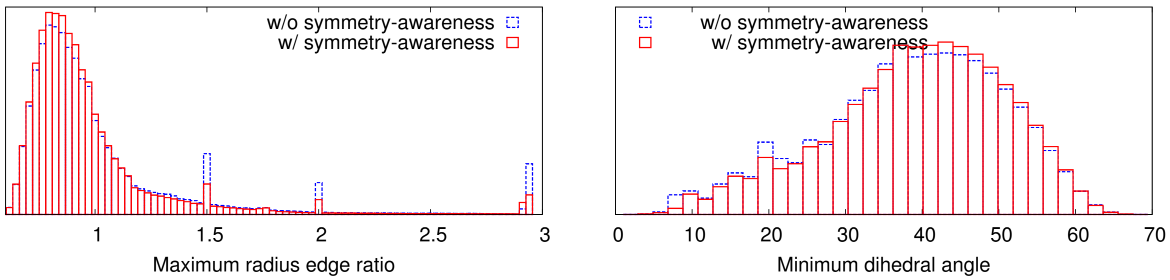
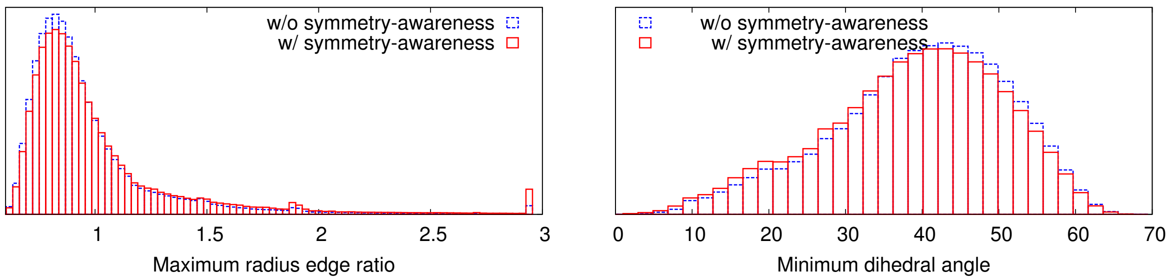
[1] http://viennamesh.sourceforge.net



(a) Objects of Category B.



(b) Objects of Category C.

Fig. 6: Average element quality histograms for maximum radius edge ratio and minimum dihedral angle.

4. Summary and Future Works

For complex objects, especially objects with very thin but large layers, the proposed symmetry-aware mesh generation pipeline yields speed-ups of 3 to 10, in some cases up to 50, for synthetic benchmarks and 22 for the practically more relevant case of TSV structures without significant reduction of element quality. Additional optimizations of the various algorithms in the pipeline to minimize overhead and to further increase the performance are feasible, e.g., by investigating parallelization aspects.

Acknowledgements

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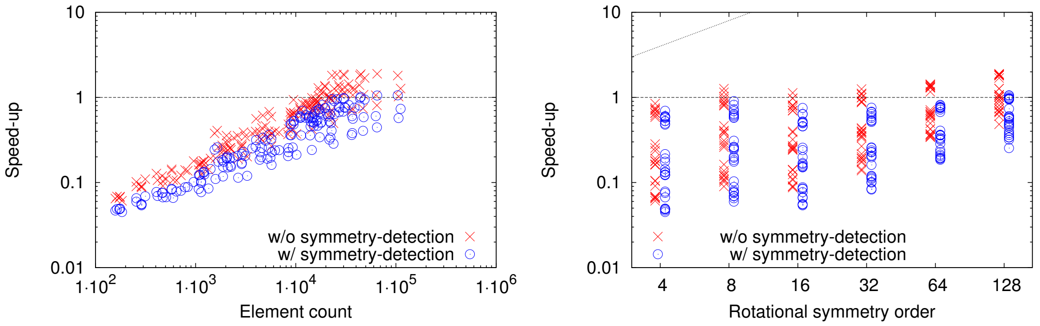
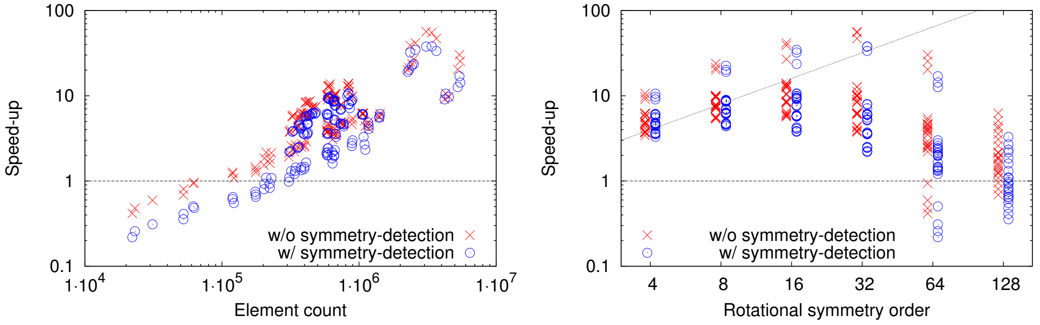
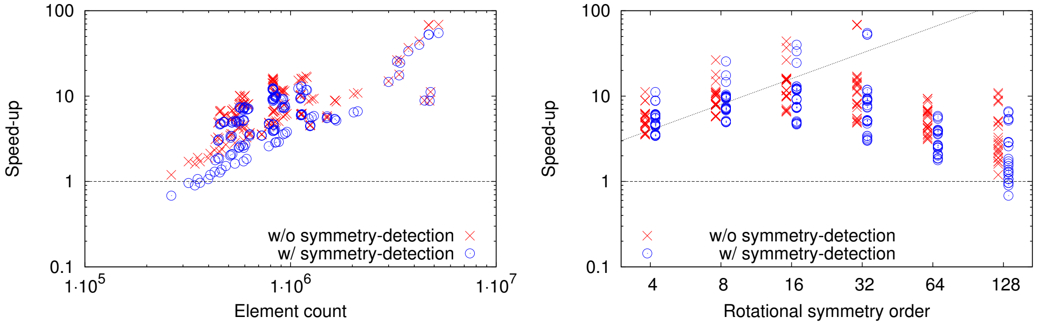


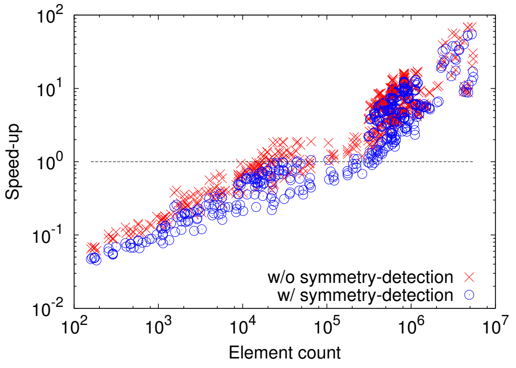
(a) Speed-ups for objects of category A.



(b) Speed-ups for objects of category B.



(c) Speed-ups for objects of category C.



(d) Speed-ups for objects of all categories.

Fig. 7: Speed-up benchmark results.