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Surface Segmentation and Polycube Construction Based on Generalized Centroidal Voronoi Tessellation

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Abstract

In this paper, we first develop a centroidal Voronoi tessellation (CVT) based surface segmentation algorithm using eigenfunctions of the Secondary Laplace operator (SLO). We then present a novel automatic polycube construction algorithm based on a generalized harmonic boundary-enhanced CVT (gHBECVT) by including the curve-skeleton information. Based on the constructed polycube, we generate quality all-hexahedral meshes in both parametric and physical domains through the parametric mapping. Several examples are presented in this paper to show the robustness of our algorithms.

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Keywords: surface segmentation; polycube; centroidal Voronoi tessellation; all-hexahedral mesh.

1. Introduction

Surface segmentation plays an important role in geometric processing whose main task is to decompose a surface mesh into meaningful parts. The secondary Laplace operator (SLO) [1] was developed based on the second fundamental form of the surface. Since its eigenfunctions reflect the curvature-related surface features, concave creases and convex ridges can be captured. The SLO was used for surface segmentation by first computing eigenfunctions and mapping vertices onto a p -dimensional space using the first p modes, and then clustering vertices into a series of groups or surface patches using Prediction Analysis for Microarrays (PAM) method [2]. The polycube construction can be built through mesh segmentation with certain geometric constraints. The harmonic boundary-enhance centroidal Voronoi tessellation (HBECVT) model [3] for automatic polycube construction extends EWCVT [4] and HEWCVT [5] in image segmentation to mesh segmentation. It takes into account the local neighbouring information of each triangle, which tends to make the boundary of the final segmentation shorter and smoother.

Inspired by these techniques, we first develop a CVT-based surface segmentation algorithm by using eigenfunctions of the SLO. We then present a novel generalized harmonic boundary-enhanced CVT (gHBECVT) model for polycube construction by using local coordinates to define the generators flexibly in the normal space with the help of the curve-skeleton information. Based on the constructed polycube, we generate quality all-hexahedral (all-hex) meshes through the parametric mapping. The key contributions of our work include: (1) CVT-based surface segmentation

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using the SLO eigenfunctions is developed, which reduces the computational cost and improves the segmentation by eliminating unsmoothed boundary and over-segmentations; (2) The gHBECVT algorithm extends HBECVT model, and improves the surface segmentation and polycube construction by reducing unnecessary singularities. By including the curve-skeleton information, the gHBECVT-based algorithm is insensitive to the shape deformation.

2. CVT-based Surface Segmentation with SLO Eigenfunctions

Several examples were presented in [1] to show the advantages of the SLO eigenfunctions using PAM for surface segmentation. However, the PAM-based surface segmentation is time consuming, and may result in jaggy boundary and over-segmented results. To generate better segmentation result, we apply the efficient CVT-based clustering idea to the SLO eigenfunctions by incorporating physical information to avoid over-segmentation and jaggy boundaries, and reduce the computational cost.

Given an input surface mesh, we first compute eigenfunctions of the SLO and map vertices onto a p -dimensional space using the first p modes. Let the dataset $VT = \{VT_i\}_{i=1}^{nv}$ denote all vertices of the surface mesh with $VT_i = (VT_{i1}, \dots, VT_{ip})^T$, where nv is the total number of vertices and VT_i represents the assigned p -dimensional vector of the i^{th} vertex $v(i)$. Let $C = \{c_l\}_{l=1}^L$ denote a set of typical p -dimensional vectors. The Voronoi region V_k ($k = 1, \dots, L$) in VT corresponding to c_k can be defined as

$$V_k = \{VT_i \in VT : \text{dist}(VT_i, c_k) \leq \text{dist}(VT_i, c_l), \text{ for } l = 1, \dots, L\}, k = 1, \dots, L, \quad (1)$$

where $\text{dist}(VT_i, c_k) = \sqrt{\|VT_i - c_k\|^2 + \eta \tilde{n}_k(v(i))}$ is a metric to calculate the distance between vector VT_i and generator c_k . In this distance metric, the first term $\|VT_i - c_k\|^2$ measures the distance in the p -dimensional eigenfunction space. The second term $\tilde{n}_k(v(i))$ introduces the neighbouring physical information which measures the number of vertices that do not belong to the k^{th} cluster within $N_\omega(v(i))$, which is a ω -rings neighboring region centered at vertex $v(i)$. η is a positive weighting factor to balance these two terms. The boundary-enhanced term $\tilde{n}_k(v(i))$ extends the idea of HBECVT [3] from triangle element to vertex, and represents the probability that vertex $v(i)$ belongs to the k^{th} cluster in physical space. $\tilde{n}_k(v(i))$ is small if the majority of neighbourhoods within $N_\omega(v(i))$ belong to the k^{th} cluster, and vice versa. The set $V = \{V_l\}_{l=1}^L$ is called a *Voronoi tessellation* of the data set VT and the set $C = \{c_l\}_{l=1}^L$ are referred to as the *Voronoi generators*. Each vertex is assigned to one of clusters, thus we have $VT = \cup_{l=1}^L V_l$ and $V_i \cap V_j = \emptyset$ if $i \neq j$. CVT-based algorithm aims at minimizing an energy function E which measures how tightly each cluster is packed until a certain criterion is met. Inspired by the HBECVT clustering energy [3] defined in the normal space for polycube construction, we can extend it to the p -dimensional vector space. Given any set of generators $C = \{c_l\}_{l=1}^L$ and any partition $U = \{U_l\}_{l=1}^L$ of VT , the clustering energy function of $(C; U)$ can be defined as

$$E(C; U) = \sum_{i=1}^{nv} \left(L \left/ \sum_{l=1}^L (\|VT_i - c_l\|^2 + \lambda \tilde{n}_l(v(i)))^{-1} \right. \right). \quad (2)$$

The updated centroid of each cluster U_k is defined to be the p -dimensional vector c_k^* which minimizes the clustering energy with respect to c_k , then we can obtain an iterative formula $c_k^* = \frac{\sum_{i=1}^{nv} \mu_{ik} VT_i}{\sum_{i=1}^{nv} \mu_{ik}}$, where $\mu_{ik} = \left(\sum_{l=1}^L \frac{\text{dist}^2(VT_i, c_k)}{\text{dist}^2(VT_i, c_l)} \right)^{-2} \cdot \mu_{ik}$ can

be viewed as a soft membership function which reflects the degree of possibility of $v(i)$ that is associated with the k^{th} cluster and how much influence $v(i)$ has in the calculation of new centroids. For an arbitrary Voronoi tessellation $(\{c_l\}_{l=1}^L; \{V_l\}_{l=1}^L)$ of VT where $V = \{V_l\}_{l=1}^L$ are the corresponding Voronoi regions associated with $C = \{c_l\}_{l=1}^L$, the clustering energy $E(C; V)$ is minimized only if $(C; V)$ form a CVT of VT , i.e., V are Voronoi regions of VT associated with the generators C and simultaneously C are the corresponding centroids of the region V .

Algorithm of SLO-CVT

Given a surface triangle mesh S , positive integer L , weighting factor η , neighbourhood size ω and error tolerance ε (e.g. $\varepsilon = 10^{-4}$), we convert S to a quadrilateral mesh M using the cross field-based surface parameterization [1]. E_i denotes the clustering energy in the i^{th} iteration. Then we perform the following:

1. Compute the SLO eigenfunctions and map vertices of M onto a p -dimensional space by selecting the first p modes. Initialize L generators $\{c_l\}_{l=1}^L$ by selecting L vertices and taking their p -dimensional eigenfunction values;

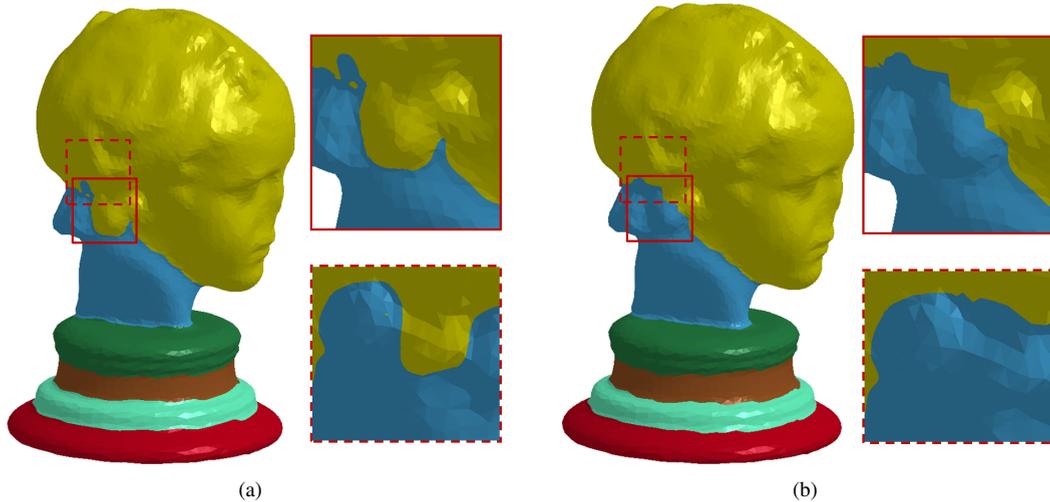


Fig. 1. Results of the bust model. (a) Segmentation result using PAM-based method and (b) Segmentation result using CVT-based method.

2. Assign each vertex to the cluster whose generator has the shortest distance to it, and determine the Voronoi cluster $\{V_l\}_{l=1}^L$ associated with generators $\{c_l\}_{l=1}^L$ by (1); and
3. Update the cluster centroids $\{c_l^*\}_{l=1}^L$ iteratively until a termination criterion such as $\frac{E_{i+1}-E_i}{E_i} < \varepsilon$ is reached. Otherwise, update generators by setting $c_l = c_l^*$ for $l = 1, \dots, L$ and return to Step 2.

Fig. 1 shows segmentation results of the bust model using both PAM method and CVT-based method from Modes 1-3 of the SLO, where neighbouring parts are rendered with different colors. We can observe that our CVT-based clustering method eliminate jaggy boundaries and over-segmentation, and the computational time is reduced from 287.6 to 2.2 seconds.

3. gHBECVT and Polycube Construction

In this section, we extend the HBECVT algorithm to gHBECVT model for automatic polycube construction. Given an input triangle mesh T , the mean curvature flow algorithm [6] is firstly applied to generate its curve-skeleton CS . Let the dataset $X = \{x_{T(i)}\}_{i=1}^n$ denote all the unit normals $x_{T(i)}$ of the triangle mesh according to the global coordinate system, where $T(i)$ represents the i^{th} triangle in the physical space. Let $CS = \{cs_j\}_{j=1}^m$ denote all curve-skeleton points. We first compute the local coordinate system of each point cs_j and also the transformation matrix Q from the global to the local coordinate system. Fig. 2 shows the input parabola model and its curve-skeleton, the local coordinate system of each curve-skeleton point can be defined using the tangent and the plane perpendicular to the tangent. As shown in Fig. 2(b), any unit vector \hat{u} in the global coordinate system $\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ can be transferred to the local coordinate system $\{\hat{e}'_1, \hat{e}'_2, \hat{e}'_3\}$ by $\hat{u}' = Q\hat{u}$, where Q is a 3×3 rotation matrix and $Q_{ij} = \cos(\hat{e}_i, \hat{e}'_j) = \hat{e}_i \cdot \hat{e}'_j$. Since each triangle can be assigned to its nearest curve-skeleton point, we can transform its normal vector in the global coordinate system to the local coordinate system by $x'_{T(i)} = Q_{T(i)}x_{T(i)}$, where $Q_{T(i)}$ is the transformation matrix between the local coordinate system $T(i)$ assigned to and the global coordinate system. Thus, we can calculate a new set of unit normals $X' = \{x'_{T(i)}\}_{i=1}^n$ from $X = \{x_{T(i)}\}_{i=1}^n$. Note that the normal vector for each triangle $T(i)$ is transformed via a rotation matrix with the help of the curve-skeleton, which makes it insensitive to the shape deformation.

Let $C = \{c_l\}_{l=1}^L$ denote a set of typical unit normal vectors. Inspired by the HBECVT model [3], we define a new generalized distance metric in the normal space together with a boundary-enhanced term in order to include local neighbouring information. For each triangle $T(i)$, we denote a local neighborhood as $N_\omega(T(i))$, which is a ω -rings neighboring region centered at triangle $T(i)$. The generalized boundary-enhanced distance between $x'_{T(i)}$ and c_k can be

defined as $dist(x'_{T(i)}, c_k) = \sqrt{\|x'_{T(i)} - c_k\|^2 + \lambda \tilde{n}_k(T(i))}$, where λ is a positive weighting factor. $\tilde{n}_k(T(i))$ represents the number of triangles that do not belong to the k^{th} cluster within $N_\omega(T(i))$. Note that the generalized distance measures the distance between the the triangle $T(i)$ and clusters in the locally transformed normal space X' instead of the global

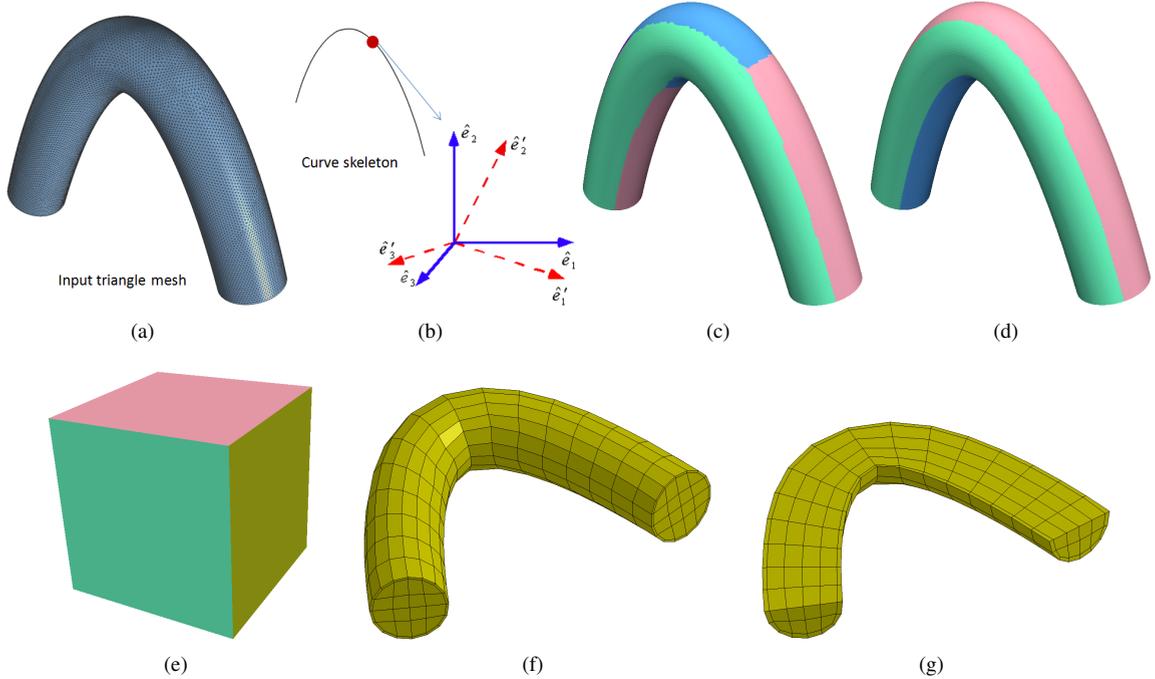


Fig. 2. Results of the parabola model. (a) Input triangle mesh; (b) curve-skeleton and local coordinate system; (c) HBECVT segmentation result; (d) gHBECVT segmentation result; (e) parametric mapping result; (f) all-hex mesh and (g) some elements are removed to show the interior of (f).

normal space X . For any set of generators $C = \{c_l\}_{l=1}^L$ and any partition $U = \{U_l\}_{l=1}^L$ of X' , we define the generalized harmonic boundary-enhanced clustering energy as

$$E^G(C; U) = \sum_{i=1}^n \left(L / \sum_{l=1}^L \left(\|x'_{T(i)} - c_l\|^2 + \lambda \tilde{n}_l(T(i)) \right)^{-1} \right). \quad (3)$$

To calculate the updated centroids $\{c_k^*\}_{k=1}^L$, we minimize E^G with respect to the generator c_k ($k = 1, \dots, L$). Then we can obtain an iterative formula as $c_k^* = \frac{\sum_{i=1}^n u_{ik}^G x'_{T(i)}}{\sum_{i=1}^n u_{ik}^G}$, where $u_{ik}^G = \left(\sum_{l=1}^L \frac{\text{dist}^2(x'_{T(i)}, c_k)}{\text{dist}^2(x'_{T(i)}, c_l)} \right)^{-2}$.

Algorithm of gHBECVT

Given a surface triangle mesh $X = \{x_{T(i)}\}_{i=1}^n$, positive integer $L = 6$, weighting factor λ , neighbourhood size ω and error tolerance ε (e.g. $\varepsilon = 10^{-4}$), we perform the following with E_i^G denoting the gHBECVT energy in the i^{th} iteration.

1. Calculate local coordinate systems for all curve-skeleton points and assign each triangle to the nearest curve-skeleton point. Then compute a new set of unit normals X' from X for each triangle $T(i)$;
2. Assign each triangle $T(i)$ to the cluster whose generator has the shortest distance to it, and determine the boundary-enhanced Voronoi clusters $\{V_l\}_{l=1}^L$ of X' associated with generators $\{c_l\}_{l=1}^L$; and
3. For each cluster V_l ($l = 1, \dots, L$), determine the cluster centroids c_l^* . If a termination criterion such as $\frac{E_{i+1}^G - E_i^G}{E_i^G} < \varepsilon$ is reached, return $(\{c_l\}_{l=1}^L; \{V_l\}_{l=1}^L)$ and exit; otherwise, set $c_l = c_l^*$ for $l = 1, \dots, L$ and return to Step 2.

Fig. 2(c,d) show the segmentation results of the parabola model using HBECVT and gHBECVT respectively. We can observe that HBECVT segments the surface in the global normal space, which introduces unnecessary singularities (16 singularities). However, our extended gHBECVT algorithm aligns to the shape deformation better by segmenting the surface in the transformed normal space, and it yields only 8 singularities in total. Inherited from the HBECVT model, our proposed gHBECVT algorithm can automatically and robustly construct polycubes for general arbitrary geometric domains. By including the curve-skeleton of the input surface, our extend gHBECVT segmenta-

tion algorithm is insensitive to shape deformation and can remove unnecessary singularities with compact polycube structure. After the gHBECVT-based mesh segmentation, the next step is to create a surface parametric mapping which aims at creating a one-to-one mapping f from the given surface T to a parameter domain T^* . Fig. 2(d) shows the parametric mapping of the parabola model. Based on the constructed polycubes, uniform all-hex meshes C can be generated by first meshing the resulting polycube with an axis-aligned grid, and then mapping it back onto the input surface via parametric mapping. Pillowing and optimization-based smoothing are implemented to further improve the mesh quality. Fig. 2(f,g) show the all-hex mesh after the quality improvement (minimal Jacobian ≥ 0.32).

4. Results and Conclusion

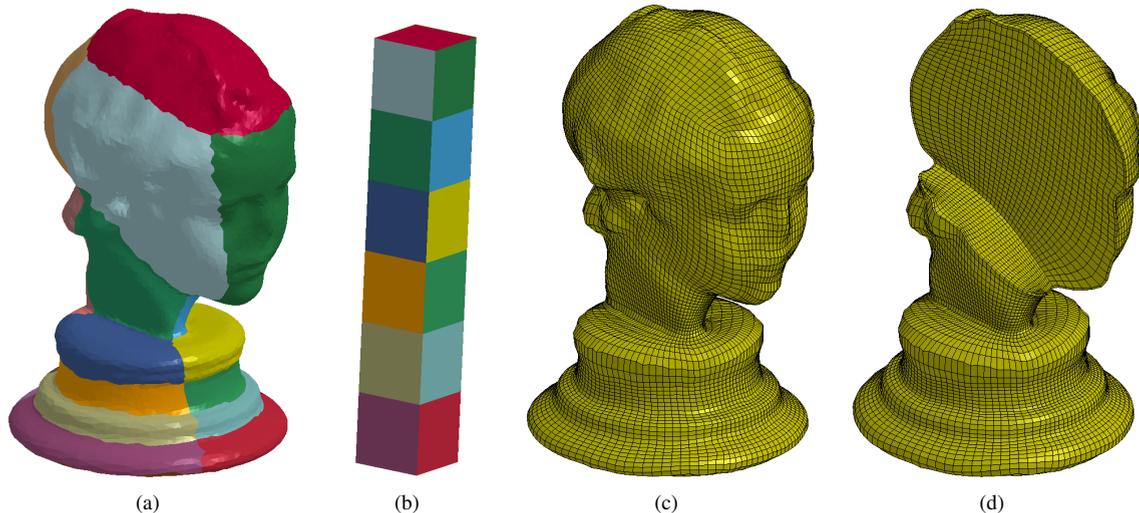


Fig. 3. Results of the bust model. (a) Segmentation result using gHBECVT-based method; (b) parametric mapping result; (c) all-hex mesh and (d) some elements are removed to show the interior of (c).

We have applied the presented algorithms to several datasets, and generated both surface segmentation results and valid polycubes with all-hex meshes. All results were computed on a PC equipped with a 2.93 GHz Intel X3470 CPU and 8GB of Memory. We take the surface segmentation using SLO-CVT as a pre-segmentation, where the surface is segmented into tubular parts. Based on the pre-segmentation, we apply gHBECVT to segment each part into multiple segments where each segment can be mapped onto one face of the resulting polycube. Fig. 3 shows the gHBECVT-based segmentation results for polycube construction and all-hex mesh of the bust model (minimal Jacobian ≥ 0.11). In conclusion, a novel CVT-based surface segmentation algorithm using eigenfunctions of the SLO is proposed, which avoids over-segmentation and jaggy boundaries, and reduces computational cost. We also present an automatic polycube construction algorithm based on gHBECVT model. Our extended gHBECVT segmentation algorithm is insensitive to shape deformation and can generate compact polycube structure by eliminating unnecessary singularities. In the future, we will apply our algorithms to more complicated models.

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