Centroidal Voronoi Tessellation based PolyCube Construction and Adaptive All-Hexahedral Mesh Generation

Kangkang Hu, Yongjie Jessica Zhang

Computational Bio-modeling Laboratory, Department of Mechanical Engineering, Carnegie Mellon University

Introduction

This poster presents a novel automatic PolyCube construction algorithm using centroidal Voronoi tessellation (CVT) based surface segmentation. Given a smooth surface triangle mesh, we segment triangles into six clusters in the surface normal space while satisfying the constraints of PolyCube construction. We develop a new harmonic boundary-enhanced CVT (HBECVT) method by including local neighbouring information in the energy function. Based on the constructed PolyCube, we then generate quality all-hexahedral (all-hex) meshes. The uniform all-hex mesh and volumetric T-mesh can be obtained through the octree subdivision and mapping. We can also generate adaptive all-hex meshes by extracting the dual mesh from a hybrid octree.

Algorithm Overview

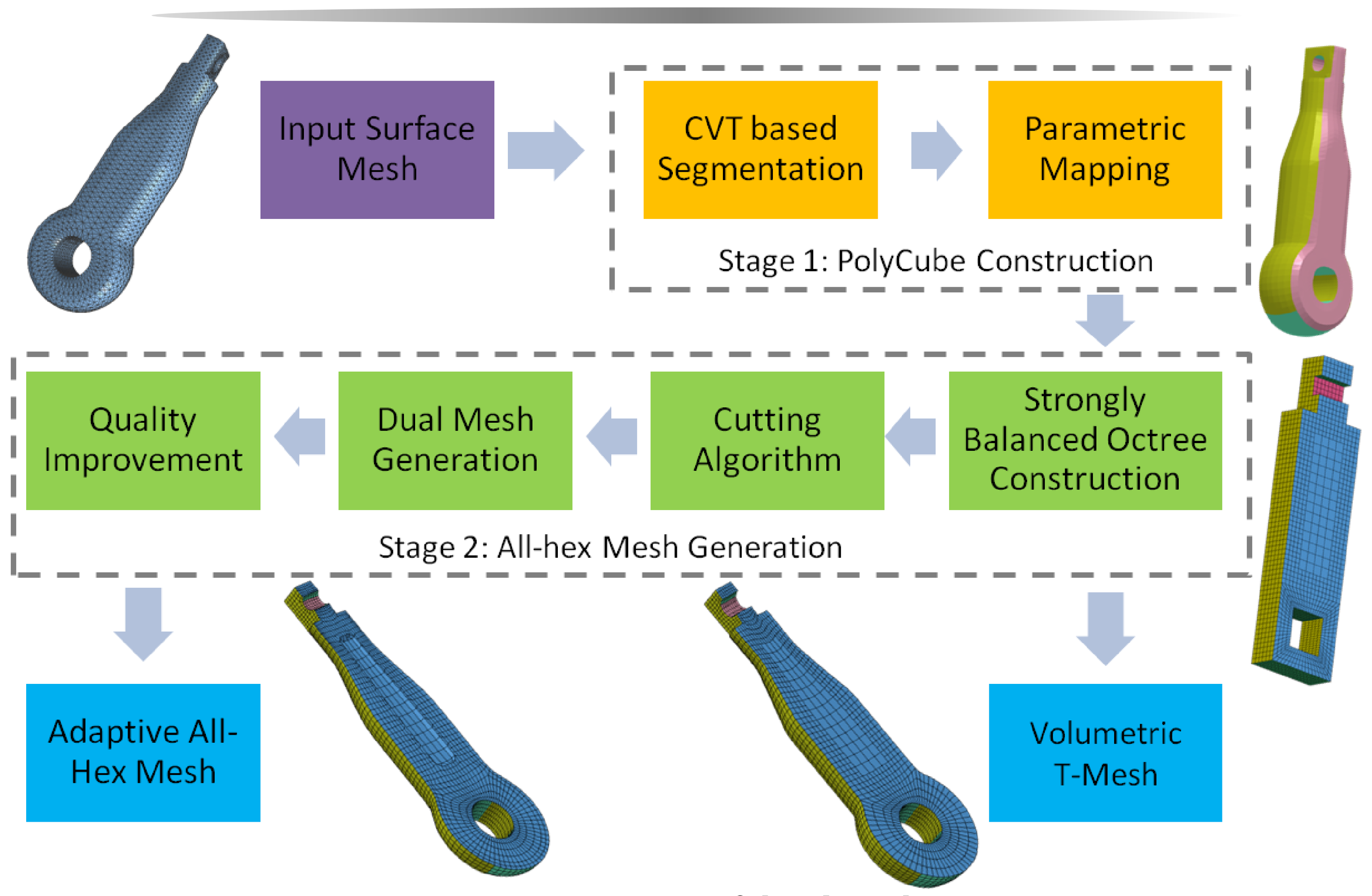

Fig. 1 An overview of the algorithm.

CVT based PolyCube Construction

➢ HBECVT based Mesh Segmentation

Given an input triangle mesh T, we partition it into six segments using CVT based segmentation. For any set of values $C=\{c_l\}_{l=1}^{L}$ and any partition $U=\{U_l\}_{l=1}^{L}$, we define the harmonic boundary-enhanced CVT (HBECVT) energy as

$$E_H(C;\mathrm{U})=\sum_{i=1}^{n}\left(L\Big/\sum_{l=1}^{L}\left(\left|x_{T(i)}-c_l\right|^2+\lambda\tilde{n}_l(T(i))\right)^{-1}\right) \quad (1)$$

Classic CVT energy: $E(C;\mathrm{U})=\sum_{l=1}^{L}\sum_{x_{T(i)}\in U_l}\left|x_{T(i)}-c_l\right|^2$

Algorithm of HBECVT

Given: an input triangle mesh T, the weighting parameter λ:

1. For each triangle *T(i)*, assign the triangle to the segment whose centroid is nearest to it.
2. For each cluster, determine the cluster means by minimizing equation (1).
3. If no transfer occurs, return and exit; otherwise, go to Step 1.

Return: six patches along with six principle directions.

The boundary-enhanced term in HBECVT takes into account the local neighbouring information of each triangle, which tends to make the boundary of the final segmentation shorter and smoother.

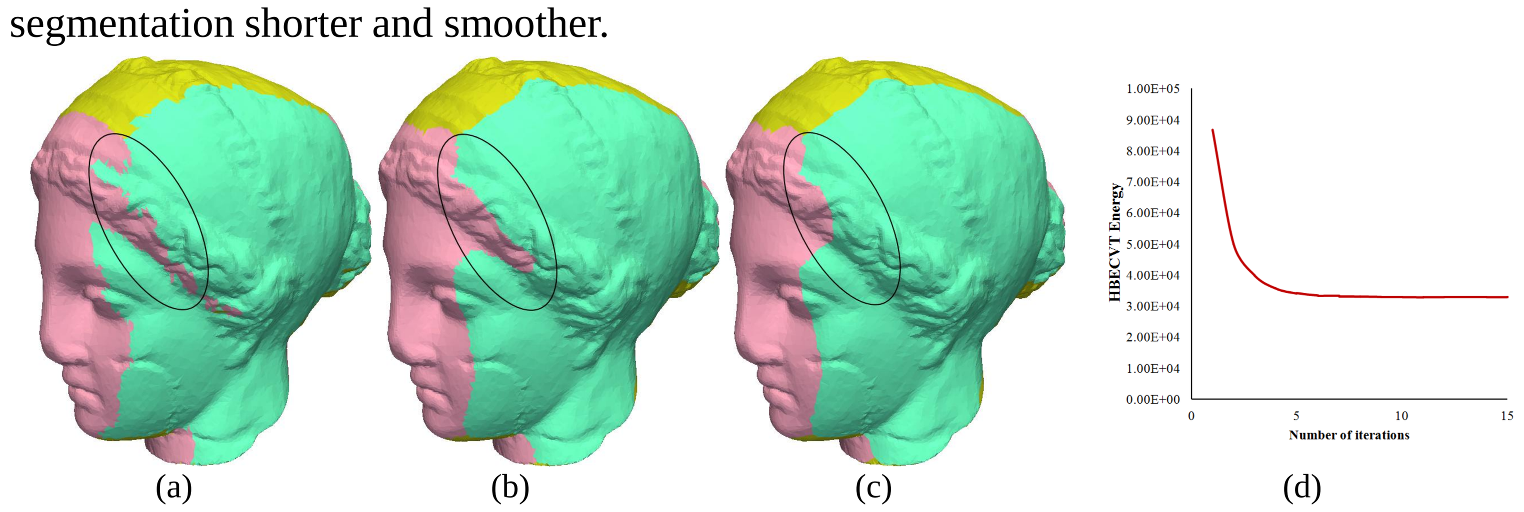

Fig. 2 Results of the igea model. (a) Classic CVT segmentation; (b-c) HBECVT segmentation after 4 and 10 iterations respectively; and (d) HBECVT energy curve.

➢ Parametric Mapping

Use planar domain parameterization to create a parametric mapping between the input triangle mesh and planar polygon of the PolyCube.

• Planar domain parameterization $\sum_{j\in n_i} w_{ij}\left(f(V_j)-f(V_i)\right)=0$ (2)

where w_{ij} is the weight and n_i is the number of vertices adjacent to V_i.

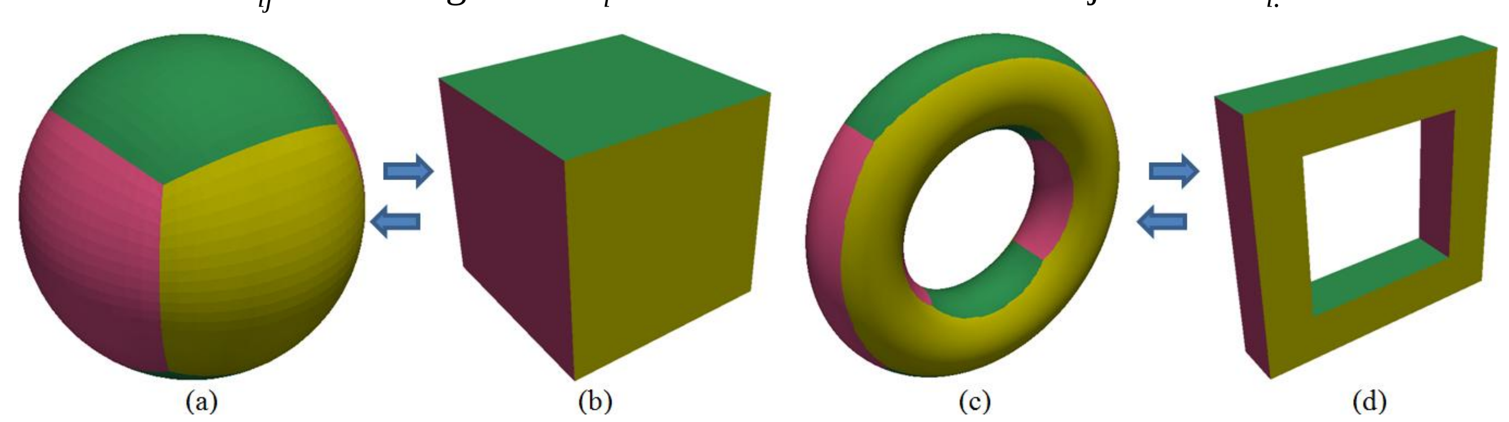

Fig. 3 Mapping of sphere and torus model.

All-hex Mesh Generation

- **Uniform all-hex mesh:** parametric mapping and interpolation.
- **Volumetric T-mesh:** refinement with a surface approximation error.
- **Adaptive all-hex mesh:** hybrid octree and dual mesh extraction.

➢ Hybrid Octree

- Five unique transition cases for the cutting procedure:
  - One transition on face (case a)
  - Four transitions on edge (case b, c, d and e)
- Each grid point in the hybrid octree is always shared by eight polyhedral cells.

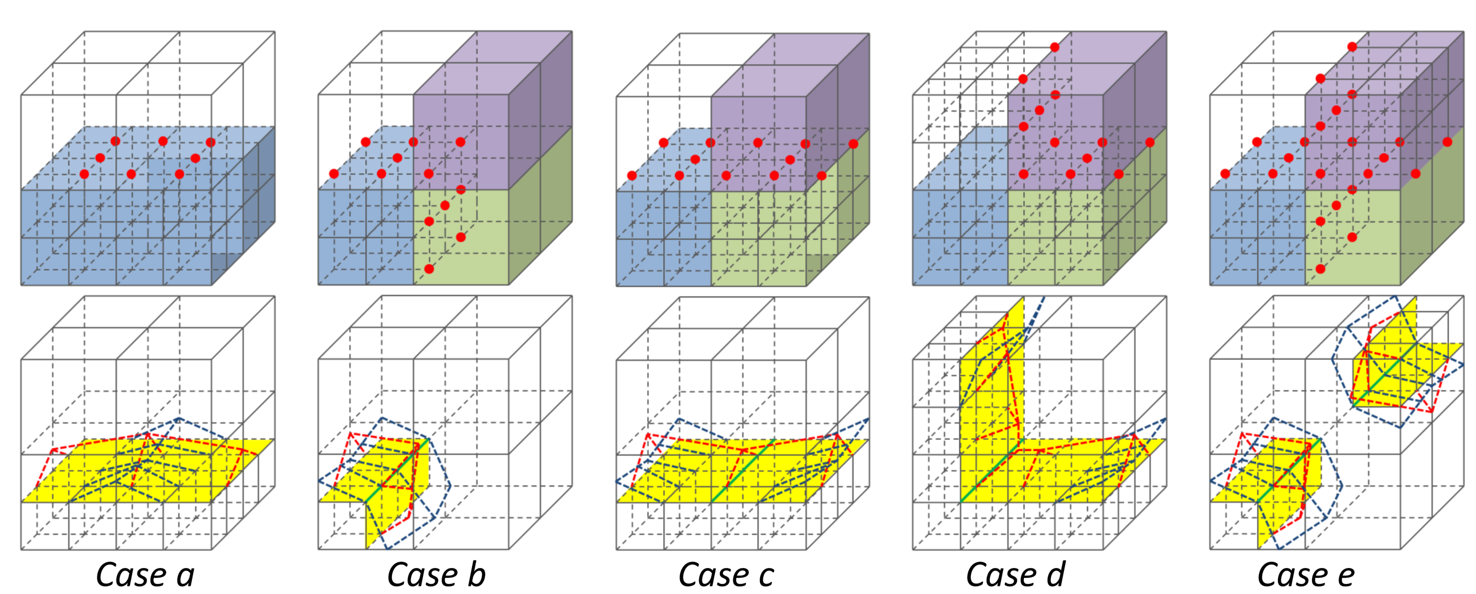

Fig. 4 First row: five unique transition cases; second row: Maréchal's cutting method; third row: our cutting method.

- In the constructed hybrid octree, all grid points are shared by eight polyhedral cells, the obtained dual mesh is all-hex.

Results and Discussions

Figs. 5-7 show results of three models. In each example, we show the HBECVT based segmentation result, corresponding PolyCube mapping result, subdivision results in both parametric domain and physical domain, and adaptive all-hex mesh result. From the results we can observe that sharp features are well preserved and the obtained adaptive meshes all have high quality (minimal Jacobian > 0.10).

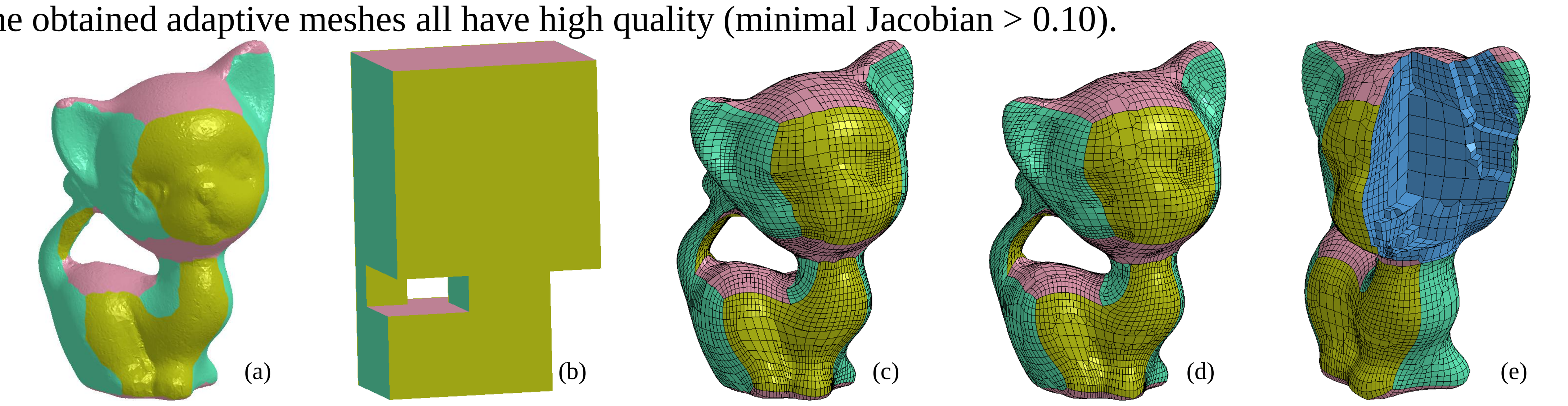

Fig. 5 Results of the kitten model. (a) HBECVT based mesh segmentation result; (b) parametric mapping result; (c) subdivision result in physical domain (volumetric T-mesh); (d) adaptive all-hex dual mesh; and (e) some elements are removed to show the interior of (d).

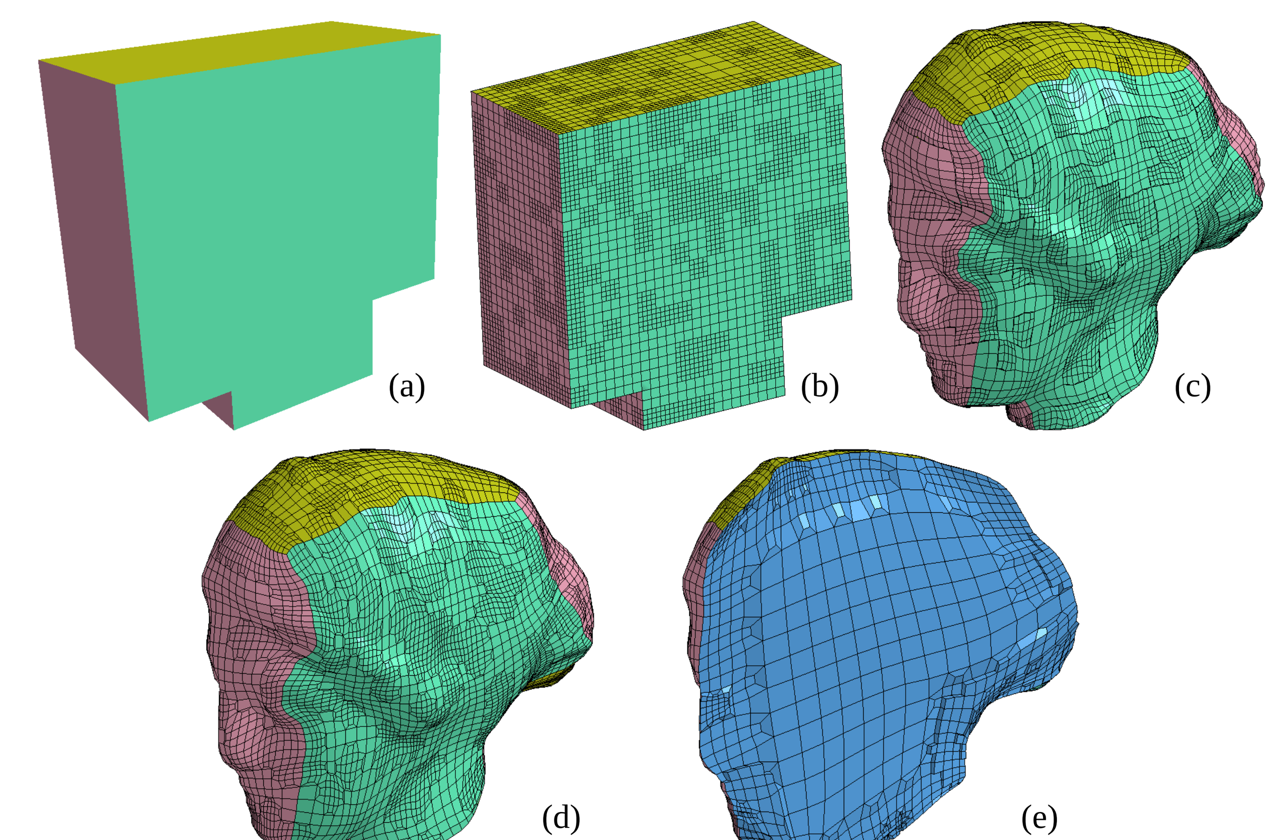

Fig. 6 Results of the igea model. (a) Parametric mapping; (b) subdivision in parametric domain; (c) subdivision in physical domain (T-mesh); (d) all-hex mesh; and (e) some elements are removed to show the interior of (d).

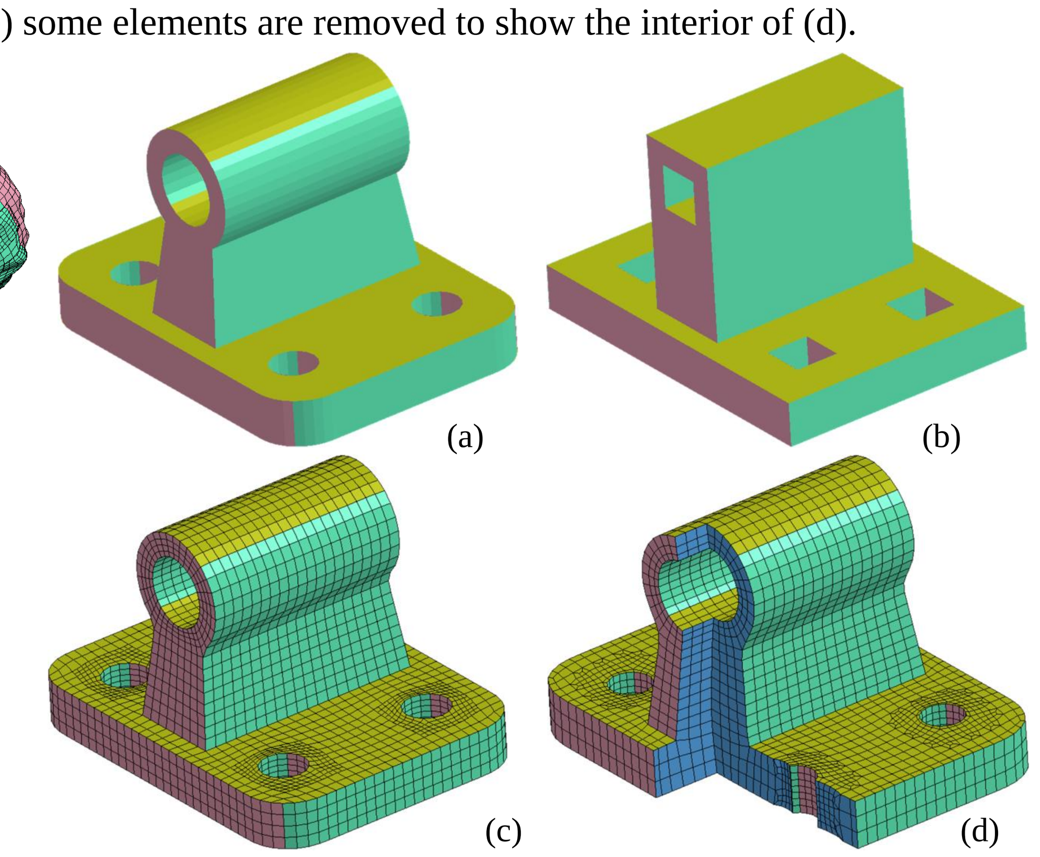

Fig. 7 Results of the base model. (a) HBECVT based mesh segmentation result; (b) parametric mapping result; (c) subdivision result in physical domain (volumetric T-mesh); and (d) adaptive all-hex mesh with some elements removed to show the interior.

Table 1: Mesh statistics of all the tested models

Model	Input mesh (vertices, elements)	Element levels	Output mesh (vertices, elements)	Jacobian (worst, best)	Jacobian (T-mesh) (worst, best)	Time (s)
Kitten	(80,000, 160,000)	3	(33,466, 30,104)	(0.10, 1.00)	(0.13, 1.00)	242.7
Igea	(4,500, 8,996)	3	(49,714, 44,586)	(0.14, 1.00)	(0.18, 1.00)	175.6
Base	(5,292, 10,600)	2	(12,720, 10,048)	(0.21, 1.00)	(0.33, 1.00)	53.8

Conclusions and Future Work

Contributions: A novel automatic PolyCube construction algorithm using HBECVT based mesh segmentation is developed. Moreover, a new mesh generation algorithm is proposed based on a hybrid octree to construct adaptive all-hex meshes with good quality. Our method can also generate T-meshes with good quality, which can be directly used to construct T-splines.

Future works: To construct solid T-spline, which can be used for the isogeometric analysis.

K. Hu and Y. Zhang were supported in part by PECASE Award N00014-14-1-0234, and NSF CAREER Award OCI-1149591.