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## Direct editing on hexahedral mesh through dual operations

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### Abstract

The finite element mesh regeneration, which is usually required during the design model verification process, is time consuming. Mesh editing is a kind of efficient mesh regeneration technique. In this paper, a direct editing method for hexahedral mesh is proposed to achieve high quality and adaptive hexahedral mesh regeneration. After the user applies direct editing on the CAD model, effective mesh deformation and optimization are automatically conducted on the associated hexahedral mesh model. To optimize the deformed mesh, firstly, simplified fundamental sheet configuration conversions are applied to improve the boundary mesh quality; secondly, dual operations are carried out to insert/extract entire sheets; thirdly, according to the measurement of the hexahedron deformation, those mesh regions whose mesh sizes become not reasonable due to the deformation are coarsened and refined through localized dual operations. The proposed method can not only effectively achieve hexahedral mesh editing, but also facilitate the interoperation of CAD model and mesh model.

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### 1. Introduction

In current product design, the product design model is frequently modified to meet requirement changes or new product development. For each modification, finite element meshes are regenerated to verify the product model in an early stage. But complete mesh regeneration for each minor modification is time consuming, especially when the product model has hundreds of features. For hexahedral meshes which are usually preferred to tetrahedral meshes in finite element analysis, some modifications on the CAD model even make it impossible to regenerate high-quality hexahedral meshes from scratch. In order to solve these problems, mesh editing techniques are studied to apply modifications on the hexahedral mesh, which improves the efficiency of mesh regeneration through mesh reuse.

Direct editing is a developing CAD model editing technique, which is able to modify the CAD model locally without knowing modeling history. When direct editing is conducted on the CAD model, mesh deformation has to be applied on the hexahedral mesh to conform to geometry changes. Mesh deformation(or mesh morphing) [1–3] is a kind of efficient mesh editing method, which only updates the locations of mesh nodes, while maintaining the mesh topology during this process. Because of the constant mesh topology, mesh elements are distorted if large deformation

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or rotation is required, resulting in poor mesh quality. Therefore, it is essential to coarsen and refine the deformed mesh, in order to adapt the mesh size to the new CAD model and improve the mesh quality. As the model is usually locally edited, the coarsening and refinement on the mesh also should be restricted in local regions.

Refinement templates [4–6] are usually used to refine the given domain of initial structured meshes. However, refinement domain control and mesh size gradient control are challenging problems. In order to resolve these issues, Borden et al. [7] and Tchou et al. [8,9] developed pillowing-based hexahedral mesh refinement method, and their methods can be applied on unstructured initial mesh. Through iteratively pillowing, the prescribed size map can be satisfied. Harris et al. [10] presented new refinement templates based on Tchou's method. Their method can locally refine the specified domain, but mesh size gradient is not well controlled, tending to degrade mesh quality.

Mesh coarsening is utilized to improve analysis efficiency through element density decreasing, which can be achieved by sheet extraction [11] on hexahedral meshes. Because sheet extraction can only remove the entire sheets in hexahedral meshes, this operation usually results in modification of the entire mesh. Shepherd et al. [12] generate localized sheets in order to extract them in the specified domain, but mesh size gradient control is still not considered.

The key of adapting the deformed mesh after CAD model modification is to specify the regions for coarsening and refinement. Sheffer et al. [13] proposed an efficient hexahedral remeshing method, that can be applied for CAD model parameter modification. They remove and split sheets according to the measurement of aspect ratio in the direction of sheet axis. While sheets are usually entirely compressed or stretched after parametric mesh deformation, localized mesh coarsening and refinement are required if sheets are partially compressed or stretched caused by CAD face rotation. Besides, different sequences of sheet operations result in different mesh qualities.

A hexahedral mesh direct editing method is presented in this paper. Its objectives are to facilitate hexahedral mesh editing by performing direct editing on CAD models and especially to optimize the edited mesh using dual operations. To achieve these objectives, after applying direct editing operations on the corresponding CAD model, deformation is firstly conducted on the hexahedral mesh. Then, the mesh quality at the boundary of the hexahedral mesh is improved through fundamental sheet insertion. In order to control mesh size gradient and to ensure mesh quality, dual operations are performed to extract and insert sheets, and the deformed mesh is iteratively and locally refined to satisfy the desired mesh size.

## 2. Dual Operations

Hexahedron inserting and removing can be achieved through dual operations, which can be used to modify the topology of hexahedral meshes based on the concept of sheet and chord. In a hexahedron, there are three sets of topologically parallel mesh edges and three sets of topologically parallel quads. By finding these topologically parallel relationships in the hexahedral mesh, sheet and chord can be defined as follows [14], as illustrated in Fig. 1(a):

- **Sheet:** a set of topologically parallel mesh edges, or the set of hexahedrons traversed.
- **Chord:** a set of topologically parallel quads, or the set of hexahedrons traversed.

Dual operations [14] including dicing, pillowing, sheet inflation, sheet extraction and chord collapse are used in our method. Dicing is a sheet insertion operation which splits a sheet into multiple sheets, see Fig. 1(b). Pillowing is another kind of sheet insertion operation. Given a set of connected hexahedrons called *shrink set*, firstly shrink the shrink set, then insert a new sheet to fill the gap between the shrink set and the rest of the mesh, see Fig. 1(c)(d). Sheet inflation is a type of more general sheet insertion operation, which inserts a new sheet through inflating a quad set which can split the mesh into several disconnected parts, see Fig. 1(e)(f). Contrary to sheet insertion operations, sheet extraction removes an entire sheet from the hexahedral mesh, see Fig. 1(g). Chord collapse removes a chord in the mesh, varying the connection relationship of the corresponding two sheets. For example, chord collapse is conducted on chord *C* in Fig. 1(a), connecting a part of sheet *A* and a part of sheet *B* to form a new sheet *A'* in Fig. 1(h), and the rest of sheet *A* and *B* also form a new sheet *B'*.

Among the above dual operations, the structured hexahedral mesh topology can be well kept through dicing and sheet extraction. Pillowing and sheet inflation are more flexible, because shrink set and quad set can be determined more freely. However, the side effect of performing pillowing and sheet inflation is that sometimes the mesh quality is degraded. Besides, chord collapse usually increases the degree of mesh edges, which also degrades the mesh quality.

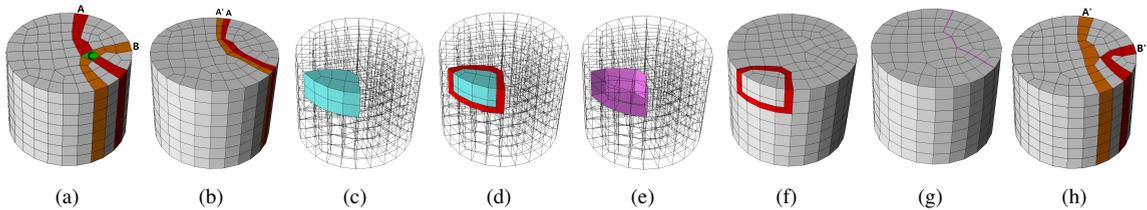


Fig. 1. Dual operations: (a) sheet and chord in hexahedral mesh; (b) dicing on sheet A; (c) shrink set; (d) pillowing on shrink set; (e) quad set; (f) sheet inflation on quad set; (g) sheet extraction on sheet A; (h) chord collapse on chord C.

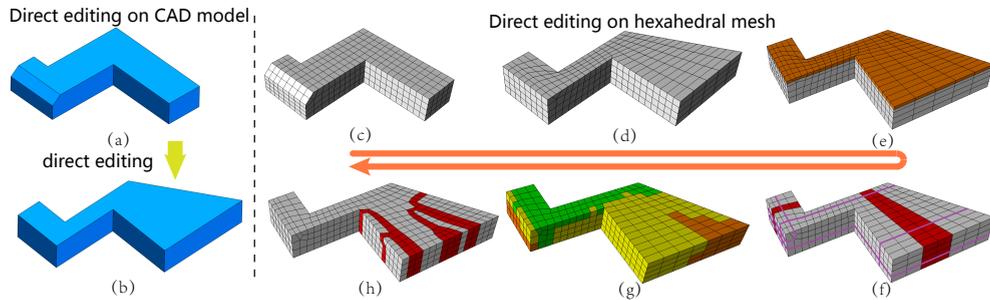


Fig. 2. Direct editing on CAD model and hexahedral mesh: (a) original CAD model; (b) CAD model after direct editing; (c) original hexahedral mesh; (d) hexahedral mesh after deformation; (e) hexahedral mesh after boundary mesh quality improvement, orange elements are newly inserted sheets; (f) mesh size control based on entire sheet insertion/extraction, red elements are newly inserted sheets, and pink lines are the boundaries of extracted sheets; (g) distribution of hexahedron deformation metric; (h) result mesh after local refinement.

### 3. Algorithm Overview

In this paper, we present an approach to achieve high quality hexahedral mesh editing. In our approach, after the user carries out direct editing on the CAD model, effective mesh deformation and optimization are automatically conducted on the associated hexahedral mesh model. In view that after the direct editing of the CAD model, the mesh model can be easily modified to keep the consistency with the CAD model through deformation, the key point here is how to optimize the quality of the deformed mesh since it is normally not good. Our strategy is to improve the quality of the deformed mesh by adding or removing certain hexahedra of the sheets and modify the topology of the hexahedral mesh using dual operations. In particular, the following two issues are addressed:

- *Evaluation of mesh quality and mesh size.* We proposed series of hexahedron deformation metrics based on dual mesh, which can evaluate the shape and size of hexahedron and sheet in the mesh.
- *Planning of dual operations.* The hexahedron deformation metrics are used to determine which dual operations should be used and the orders of applying them, and these dual operations are iteratively conducted to improve mesh quality and control the mesh size.

Based on the above ideas, our algorithm mainly consists of the following three steps as illustrated in Fig. 2:

1. Use Laplacian smoothing to deform hexahedral mesh after direct editing on the CAD models is performed;
2. Improve boundary mesh quality by fundamental sheet insertion;
3. Use dual operations to coarsen and refine the mesh regions according to the hexahedron deformation metrics.

### 4. Mesh Deformation Based on Direct Editing of CAD Model

It's intuitive and convenient for designers to locally modify the CAD model using direct editing. After designers manually modified the CAD model, in order to quickly get a mesh that conforms to the model's new geometry, the

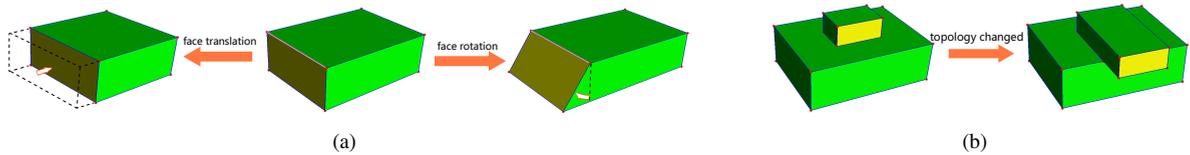


Fig. 3. Direct editing: (a)face translation and face rotation; (b)topology changed after face translation.

associated hexahedral mesh is automatically deformed using smoothing methods, which only change the positions of mesh nodes while keeping the topology unchanged. To facilitate the interoperability between the CAD model and the mesh model, we establish the mapping between the mesh boundary nodes and the geometric faces of the CAD model.

#### 4.1. Direct Editing on CAD Model

Different from traditional parametric based modification method, direct editing is a face-oriented modification method, which is able to locally modify the CAD model without updating the entire model. The basic direct editing operations include face translation and face rotation, as illustrated in Fig. 3(a). Arbitrary face translation and face rotation may change the topology of the CAD model, which may merge old faces and create new ones, as shown in Fig. 3(b). In our algorithm, since the mapping between mesh nodes and faces are fixed, therefore only direct editing operations that do not change the model topology are allowed.

#### 4.2. Mesh Deformation

The hexahedral mesh is deformed after performing direct editing on CAD models. We use Laplacian smoothing [1, 15] to perform the deformation. In order to ensure boundary constraints, the mapping between mesh boundary nodes and geometric faces of CAD models is established and maintained during mesh deformation.

We firstly move mesh boundary nodes to the new positions according to the mapping, then smooth the rest of the nodes that are not mapped. This procedure can improve mesh quality by rectifying most of the inverted mesh hexahedra, while it is still difficult to eliminate inverted hexahedrons at concave boundaries.

### 5. Boundary Mesh Quality Improvement

Some geometric curve's dihedral angles change during direct editing when their incident faces are rotated. After mesh deformation, as the topology of the mesh keeps the same, hexahedra near these curves may become poorly shaped or even tangled.

Ledoux et al.[16] proposed a method to improve the mesh quality near the mesh boundary by inserting fundamental sheets which capture geometric faces. In their method, every geometric curve can be captured by a reasonable configuration of several fundamental sheets. However this method is not suitable in our situation since modification of the mesh is local while fundamental sheet configurations are always computed for the entire mesh. Time is wasted on working out global configurations and redundant fundamental sheets are usually inserted.

In order to solve that problem, we propose simplified fundamental sheet configuration conversion rules. In [16], geometric curves are labeled from 1 to 4 according to curve's dihedral angle, and each kind of label is assigned with an ideal fundamental sheet configuration, which is varied when the curve's dihedral angle is changed. According to the geometric curve label changes, we convert local configurations using the following rules as illustrated in Fig. 4:

- **Label 1 to label 2:** insert a level 1 or level 2 fundamental sheet to capture curve's incident geometric faces;
- **Label 2 to label 1:** insert two level 1 or level 2 fundamental sheets to capture curve's incident geometric faces;
- **Label 2 to label 3:** insert a level 1 or level 2 fundamental sheet to capture curve's incident geometric faces;
- **Label 3 to label 2:** insert a level 1 or level 2 fundamental sheet to capture curve's incident geometric faces.

After applying these conversion rules, the related curves are captured by ideal fundamental sheet configurations, while the number of chords related to the other curves are not changed. The boundary mesh quality improvement for

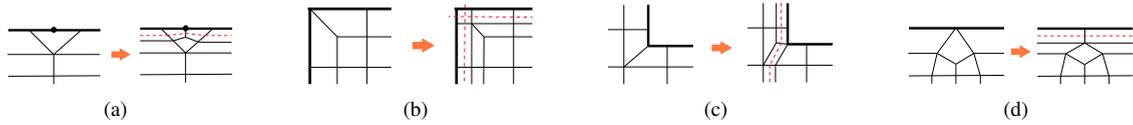


Fig. 4. Fundamental sheet configuration conversions: (a)label 1 to label 2; (b)label 2 to label 1; (c)label 2 to label 3; (d)label 3 to label 2.

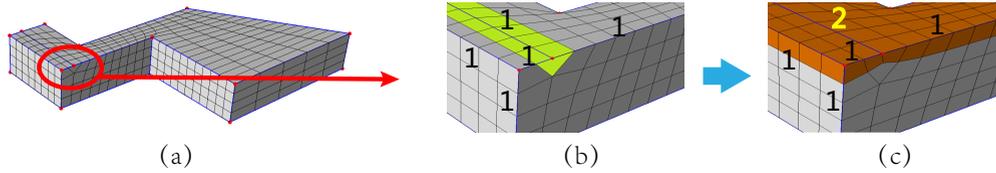


Fig. 5. Boundary mesh quality improvement: (a)deformed mesh; (b)prism-shaped hexahedra; (c)mesh quality improved.

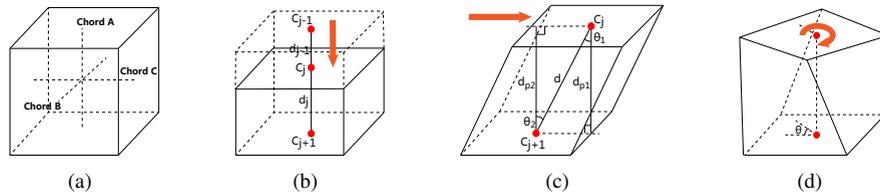


Fig. 6. Hexahedron deformation metrics: (a)hexahedron without deformation; (b)stretch deformation; (c)shear deformation; (d)twist deformation.

the example in Fig. 2(e) is illustrated in Fig. 5. In this case, prism-shaped hexahedra appear in the deformed mesh due to face rotation. Therefore we insert a level 1 fundamental sheet at the mesh boundary, converting the configuration of fundamental sheets from label 1 to label 2.

## 6. Mesh Coarsening and Refinement

In order to obtain adaptive meshes, automated mesh coarsening and refinement are conducted on the deformed meshes. According to Section 2, sheet extraction is applied on the entire sheet which may affect the whole mesh, while sheet insertion can be restricted within a local area. Therefore strategically, we firstly carry out coarsening and refinement for entirely deformed sheets using dicing and sheet extraction, operations that tend to keep the mesh well structured, then coarsen local regions by localized sheet extraction, and at last refine local regions using pillowing and sheet inflation. In these procedures, we propose series of hexahedron deformation metrics to determine how to insert/extract sheets.

### 6.1. Hexahedron Deformation Metrics

The target of our mesh coarsening and refinement is to narrow the differences of size and shape between the deformed hexahedron and the ideal one. There are three types of hexahedron deformation including stretching, shearing and twisting, see Fig. 6. We introduce several metrics to measure these types. In our algorithm, hexahedron twist deformation which does not usually occur is not currently considered.

**Definition 1 (Chord stretch metric).** *The scaled total difference between actual chord length and ideal chord length. The total difference is divided by the average mesh size of the chord, as defined in Eq. (1).*

$$T_c = \frac{\sum_{j=1}^{n_c} (d_j - h_j)}{\frac{1}{n_c} \sum_{j=1}^{n_c} h_j} \tag{1}$$

The chord stretch metric( $T_c$ ) for a given chord is defined as illustrated in Fig. 6(b),  $n_e$  is the number of hexahedra in the chord,  $d_j$  is the length of  $C_j C_{j+1}$ , which is the distance between the center point of the  $j$ th quad and the  $(j+1)$ th quad along the chord, and  $h_j$  is the ideal mesh size at the center position of  $C_j C_{j+1}$ , which is calculated before mesh deformation or predefined by users. Chord stretch metric shows the average influence of inserting/removing a hexahedron into/from the chord. We say the chord is over compressed if  $T_c \in (-\infty, -1)$ , then one or more hexahedron should be removed; the chord is over stretched if  $T_c \in (1, +\infty)$ , then one or more hexahedron should be inserted. The chord stretch metric is reasonable iff  $T_c \in [-1, 1]$ .

**Definition 2 (Hexahedron stretch complexity).** *The number of chords that satisfy  $T_c \in (-\infty, -1) \cup (1, +\infty)$  and pass through the given hexahedron.*

There are three chords passing through a hexahedron, so the range of hexahedron stretch complexity is  $TC_h \in \{0, 1, 2, 3\}$ .

**Definition 3 (Quad inflation metric).** *The average inflation ratio of the internal quad's adjacent hexahedra, and the inflation ratio is computed along the direction of quad normal, as defined in Eq. (2).*

$$I_q = \frac{d_1 + d_2}{h_1 + h_2} \quad (2)$$

Here  $I_q$  is the quad inflation metric for a given quad,  $d_1$  and  $d_2$  are the distances between the quad and its two neighboring topological parallel quads at their center points.  $h_1$  and  $h_2$  are the ideal mesh sizes at the center positions of the two hexahedra. As illustrated in Fig. 6(b),  $d_1$  and  $d_2$  are  $C_{j-1} C_j$  and  $C_j C_{j+1}$ .

**Definition 4 (Quad-pair shear metric).** *The scaled shear angle of a pair of topologically parallel quads in the hexahedron, as defined in Eq. (3).*

$$S_{qp} = \frac{1}{\pi} (\arccos \frac{d_{p1}}{d} + \arccos \frac{d_{p2}}{d}) \quad (3)$$

Here  $S_{qp}$  is the quad-pair shear metric for a given quad-pair,  $d_{p1}$  and  $d_{p2}$  are the perpendicular distances from the center point of each quad to the other quad, and  $d$  is the distance of two center points, as illustrated in Fig. 6(c).

The above metrics can be used to evaluate the deformation of the hexahedron. Furthermore, metrics are defined on sheet and quad set as follows to provide essential guidance for applying dual operations.

**Definition 5 (Sheet inflation metric).** *The sum of chord stretch metrics of the over stretched chords passing through the sheet.*

**Definition 6 (Sheet compression metric).** *The sum of chord stretch metrics of the over compressed chords passing through the sheet.*

**Definition 7 (Sheet shear metric).** *The maximum shear metric of the quad-pairs connected by the mesh edges of the sheet, that is  $S_s = \max\{S_{qp} | i = 1, 2, \dots, n_{qp}\}$ , where  $n_{qp}$  is the number of quad-pairs.*

**Definition 8 (Quad set inflation metric).** *The sum of stretch metrics of the over stretched chords passing through the quad set.*

In the process of mesh coarsening and refinement, we reduce the hexahedron stretch complexity of the mesh and control the shear metrics of the hexahedron by dual operations, ensuring that most of the hexahedra are rectified to be closer to the ideal shape.

## 6.2. Coarsening and Refinement on Entirely Deformed Sheets

In order to keep a mesh as structured as possible, we conduct dicing and sheet extraction followed by mesh smoothing to refine and coarsen the hexahedral mesh, and the hexahedron deformation metrics are recalculated after each individual dual operation. In this step, only entirely deformed sheets are diced or extracted, and the partially deformed sheets are dealt with in the next step. We first determine sheet candidates for dicing and sheet extraction, then plan the order of these dual operations.

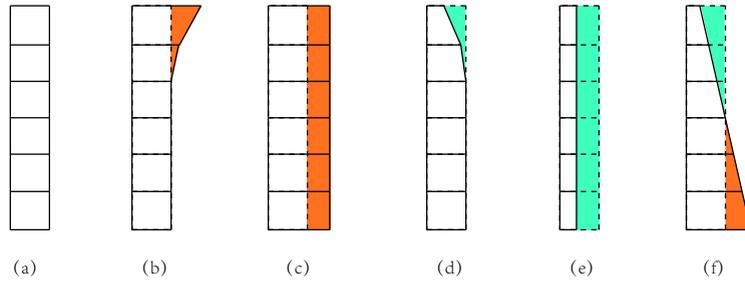


Fig. 7. Sheet deformation classification: (a) non-deformation sheet; (b) partially inflated sheet; (c) entirely inflated sheet; (d) partially compressed sheet; (e) entirely compressed sheet; (f) complicatedly deformed sheet.

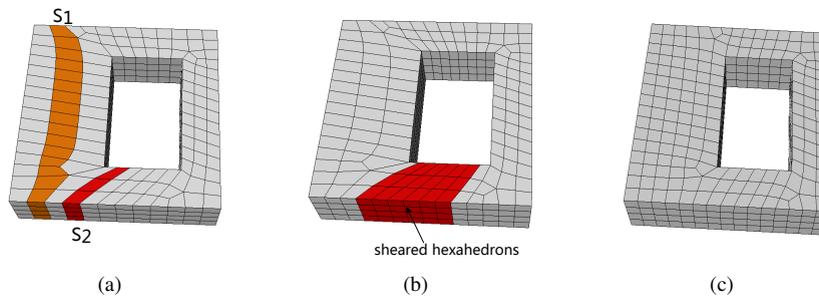


Fig. 8. Influence of dual operation order: (a) hexahedral mesh before dual operation; (b) dicing on  $S_2$ ; (c) dicing on  $S_1$ .

**Candidate Sheet Determination.** Candidate sheets for dual operations are determined for effective mesh size control. Firstly entirely deformed sheets are determined according to two metrics for a given sheet:  $p^+$  which is the ratio of the number of the over stretched chords ratio within the chords passing through the sheet; and  $p^-$  which is the ratio of the number of the over compressed chords within the chords passing through the sheet. *Entirely deformed sheet* includes *entirely inflated sheet* ( $p^+ \geq \overline{p^+}$  and  $p^- = 0$ ) and *entirely compressed sheet* ( $p^+ = 0$  and  $p^- \geq \overline{p^-}$ ). Here  $\overline{p^+}$  and  $\overline{p^-}$  are the thresholds of over stretched chords ratio and over compressed chords ratio, which are set to 0.8 in our implementation. So other deformed sheets are called *partially deformed sheet*, including *partially inflated sheet* ( $0 < p^+ < \overline{p^+}$  and  $p^- = 0$ ), *partially compressed sheet* ( $p^+ = 0$  and  $0 < p^- < \overline{p^-}$ ) and *complicatedly deformed sheet* ( $p^+ > 0$  and  $p^- > 0$ ). The classification of deformed sheets is illustrated in Fig. 7.

Based on the above classification of deformed sheets, only entirely deformed sheets are included in the candidate sheets. Besides, only the average influence of dual operations on the chords passing through the sheet are considered. For the quads on the two sides of the sheet, since their neighboring hexahedra are locally compressed or stretched, some hexahedra are possibly over compressed or over stretched. In order to avoid quad inflation metric less than 1 after dicing, candidate sheets for dicing must satisfy that  $\min\{I_{q_i}\} > 1$ . Here  $I_{q_i}$  is the quad inflation metric of the quad on the two sides of the sheet. Furthermore, fundamental sheets are not extracted, so that the geometric faces and curves can still be captured. Above all, candidate sheets for dual operations are selected based on the following rules:

- **Candidate sheet for dicing:** sheet that satisfy  $p^+ \geq \overline{p^+}$  and  $p^- = 0$  and  $\min\{I_{q_i}\} > 1$
- **Candidate sheet for sheet extraction:** sheet that is not a fundamental sheet satisfying  $p^+ = 0$  and  $p^- > \overline{p^-}$ .

**Dual Operation Order Planning.** Different orders for applying dual operations usually result in different meshes, sometimes may even degrade into poor quality. As shown in Fig. 8, if carrying out dicing on sheet  $S_2$ , the number of over sheared hexahedra increases as shown in (b). If dicing is carried out on sheet  $S_1$  whose sheet inflation metric is greater than  $S_2$ , there is no over sheared hexahedron in the result mesh, as illustrated in (c). We plan the order of dual operations according to sheet inflation metric and sheet compression metric:

1. Determine candidate sheet for dicing or sheet extraction, and terminate the algorithm if no candidate sheet exists.

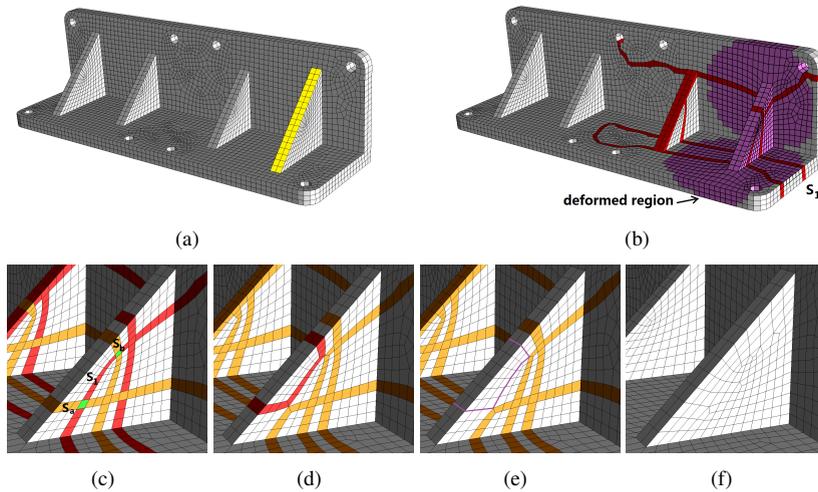


Fig. 9. Local coarsening on partially deformed sheet: (a)original mesh; (b)deformed mesh with a rib shrunk and its deformed region(purple); (c)partially deformed sheet  $S_1$  and two sheets  $S_a$  and  $S_b$  intersected with  $S_1$ ; (d)chord collapse on two chords of sheet  $S_1$  shared with  $S_a$  and  $S_b$ ; (e)sheet  $S_1$  is locally extracted; (f)partially deformed sheets in deformed region are locally extracted.

- Sort candidate sheets according to the value of its sheet inflation metric and sheet compression metric, and carry out dual operation on the candidate sheet with the maximal value, return to step 1.

After each individual dual operation is carried out, the mesh is smoothed and the hexahedron deformation metrics are recalculated so as to sort the candidate sheet according to updated metrics. After coarsening and refinement on the selected sheets, there are only non-deformation sheets and partially deformed sheets left in the mesh.

### 6.3. Local Coarsening on Partially Deformed Sheets

Our local coarsening refers to locally extracting the compressed parts of the partially deformed sheets. Direct editing operations are usually locally applied on the CAD model, while sheet extraction removes the entire sheet including the non-deformed parts. Shepherd et al. [12] proposed a local coarsening method on the specified region, and sheet extraction is localized in the specified region. Based on their idea, we select the partially deformed sheets, and try to extract only the compressed parts in the local regions.

#### 6.3.1. Partially Deformed Sheet Selection

In order to coarsen the hexahedral mesh locally, we firstly have to determine the local region where partially deformed sheets need locally extracting. During mesh deformation, only nodes of local mesh regions are remarkably repositioned, so we denote this type of mesh region as *deformed region*, where local coarsening is performed.

For the partially deformed sheets in the deformed region, they are sorted in a partially deformed sheet queue ( $Q_{pds}$ ) to select the most effective sheets for localized extraction. Here the partially deformed sheets are sorted according to the greater values of their sheet inflation metric and sheet compression metric by descending order, and only the deformed hexahedra in the deformed regions are considered for computing these metrics. For instance, we shrink one of the ribs through face translation in Fig. 9(a). Then the region with node repositioning ratio greater than 10% is determined as the deformed region, see Fig. 9(b). In the partially deformed sheet queue, we can see the first sheet  $S_1$  is very complex and goes beyond the deformed region, so its partially compressed part has to be locally extracted.

#### 6.3.2. Detailed Procedures of Localized Extraction

For partially deformed sheets, extracting is only required on its compressed parts. Therefore we use the existing sheets in the deformed region to localize the extraction of the compressed parts as much as possible, in order not to introduce excessive hexahedra in each iteration. Detailed process is as follows:

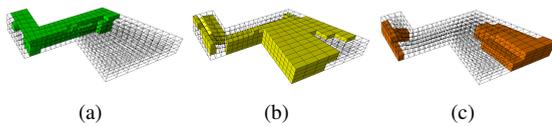


Fig. 10. Refinement regions: (a)region not to be refined; (b)simple refinement region; (c)complex refinement region.

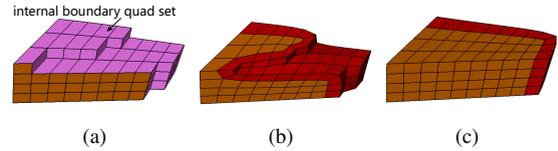


Fig. 11. Shrink set optimization: (a)initial shrink set; (b)pillowing on initial shrink set; (c)pillowing on optimized shrink set.

1. Sort the partially deformed sheets in queue  $Q_{pds}$ , and get the first sheet  $S_1$ .
2. If  $S_1$  is completely in the deformed region, extract it and go to step 1.
3. Check  $Q_{pds}$  by its order until sheet  $S_a$  is found, that  $S_a$  is intersected with  $S_1$  at chord  $C_1$ , and  $S_1$  can be converted to an entirely compressed sheet in the deformed region after collapsing  $C_1$ . Once  $S_a$  is found, extract the converted sheet after chord collapse, smooth the mesh and recalculate the related metrics, then go to step 1.
4. Check  $Q_{pds}$  by its order until a pair of sheets  $(S_a, S_b)$  is found, that  $S_a$  and  $S_b$  are respectively intersected with  $S_1$  at chords  $C_1$  and  $C_2$ , and  $S_1$  can be converted to an entirely compressed sheet in the deformed region after collapsing  $C_1$  and  $C_2$ . Once  $(S_a, S_b)$  is found, extract the converted sheet after chord collapse, smooth the mesh and recalculate the related metrics, then go to step 1, see Fig. 9(c-e).
5. Go to step 1 if  $S_1$  is not the last sheet in  $Q_{pds}$ .
6. If no sheet in  $Q_{pds}$  can be locally extracted in the previous steps, we use the method described in [12] to pillow the deformed region and insert a sheet at the boundary of the region, then collapse the chords which are the intersections between the sheets in  $Q_{pds}$  and the newly inserted sheet, so as to localize these sheets in the deformed region and then extract them, smooth the mesh and recalculate the related metrics. In this step, sheets are extracted by pairs until no sheet is partially compressed.

#### 6.4. Local Refinement on Refinement Regions

Finally, local regions that include partially inflated sheets are refined using pillowing and sheet inflation. Due to its importance, the refinement region and the order of dual operations are carefully determined. No hexahedron is removed from the mesh during the refinement process, therefore the refinement is bound to be terminated till no refinement region can be effectively refined. The procedures of local refinement are as follows:

1. Determine refinement regions and put them in a priority queue.
2. Refine the refinement region with the highest priority in the queue. If the region is successfully refined, clear the queue and return to step 1; if refinement failed, try other refinement regions in the queue. The algorithm is terminated when the queue is empty.

##### 6.4.1. Refinement Region Determination

Refinement region is a set of hexahedra that connect to each other through quads and share the same hexahedron stretch complexity. The complexity of the refinement region is the hexahedron stretch complexity of its hexahedra. Refinement region is classified into two groups according to its complexity( $C_r$ ): simple refinement region if  $C_r = 1$ ; complex refinement region if  $C_r > 1$ . Fig. 10 shows refinement regions of the example in Fig. 2(g).

Then we determine the priorities of refinement regions. Because the neighboring hexahedra of refinement region with high complexity are usually over stretched, refinement on complex refinement region also decreases the deformation complexities of its neighboring hexahedra. Therefore, the larger its complexity is, the higher the priority of the refinement region is. Besides, refinement regions should not be too small since it is hard to avoid hexahedra being over compressed within such a small area. In order to effectively refine local regions, the refinement region with the highest priority is selected for refinement: select refinement region with the highest complexity for refinement, and select refinement region with the most hexahedra if existing more than one region with the same highest complexity.

#### 6.4.2. Complex Refinement Region Refinement

Complex refinement region is stretched in multiple directions, therefore we use pillowing and sheet inflation to refine the whole region. Besides, the shrink set is optimized for mesh quality when using pillowing, and quad set is carefully selected when using sheet inflation.

*Shrink Set Optimization.* When using pillowing on complex refinement region, the shrink set is the set of hexahedra in the region. The mesh quality is usually poor if the surface of the shrink set is wrinkled, hence the shrink set has to be optimized to ensure mesh quality, as shown in Fig. 11. When carrying out pillowing, only shrink set's internal boundary quad set (internal quad set at the boundary of the shrink set) is inflated, otherwise geometric faces can not be captured by fundamental sheets.

The shrink set is optimized through hexahedron set modification, and the original hexahedra are kept as much as possible to have the refinement region refined effectively. Based on observations, thin-wall boundaries, corner boundaries, slot boundaries and step boundaries result in highly wrinkled internal boundary quad set. The rules of shrink set optimization are listed as follows:

- For a hexahedron in the shrink set, if it has a pair of quads which are both in the internal boundary quad set, this hexahedron is a thin-wall hexahedron and it is excluded from the shrink set.
- For a hexahedron out of the shrink set, if it has at least three quads in the internal boundary quad set, this hexahedron is a corner hexahedron and it is included in the shrink set.
- For a hexahedron out of the shrink set, if it has a pair of quads which are both in the internal boundary quad set, this hexahedron is a slot hexahedron and it is included in the shrink set.
- For a hexahedron out of the shrink set, if there are two hexahedra in the shrink set neighboring it, and any of the two hexahedra has two quads which are both in the internal boundary quad set, this hexahedron is a step hexahedron and it is included in the shrink set.

If existing over compressed hexahedra after region refinement, we say the refinement region is over refined and the refinement is failed. The refinement capability of shrink set is measured using *shrink ratio*:

**Definition 9 (Shrink ratio).** *The ratio of average hexahedron size after and before pillowing on shrink set.*

Shrink ratio reflects the impact of pillowing on mesh size variation. Ideally, half of the newly inserted sheet after pillowing is inside the shrink set region (the region of the shrink set before pillowing), and the other half is in the rest of the mesh. In this situation, we say that the number of hexahedra in the shrink set region is increased by half of the number of quads at the internal boundary, so we estimate shrink ratio using the following equation:

$$R_{shrink} = \frac{n_q}{2n_e} + 1 \quad (4)$$

Where  $n_q$  is the number of quads in the internal boundary quad set, and  $n_e$  is the number of hexahedra in the shrink set. Pillowing is not conducted on the shrink set if shrink ratio falls below the threshold. Because only one hexahedron is inserted into every chord passing through the shrink set on average, the shrink ratio threshold of complex refinement region is set to 0.4.

*Quad Set Selection.* Hexahedra in the complex refinement region are compressed in all directions after pillowing, so it is possible to have hexahedra over compressed at some directions. Therefore sheet inflation is carried out instead of pillowing. Besides, the internal boundary quad set of the refinement region is not always connected, so only the best self-connected quad subset is selected for sheet inflation. In order to avoid over refining the region, the ratio of over stretched chords ( $p^+$ ) passing through the candidate quad set is required to be over the threshold ( $\bar{p}^+$ ), and the candidate quad set with the greatest sum of chord stretch metric is selected for sheet inflation, then smooth the mesh and recalculate the related metrics for the other candidate quad sets. Refinement on the complex refinement region fails if no candidate quad set exists.



Fig. 12. Refinement on simple refinement region: (a)partially inflated part of the sheet; (b)refinement through pillowing on partially inflated part.

### 6.4.3. Simple Refinement Region Refinement

In order to prevent simple refinement region from being over refined, pillowing is conducted on part of the hexahedra of the region. The simple refinement region is composed of several contacting partially inflated sheets, so the partially inflated part of one of these sheets is refined using pillowing. The refinement process is as follows:

1. Select the partially inflated sheet with the highest priority in the simple refinement region.
2. Set the partially inflated part of the selected sheet as the shrink set, and optimize the shrink set.
3. Carry out pillowing on the shrink set, smooth the mesh and recalculate the related metrics.

*Partially Inflated Sheet Selection.* Then we measure the priority of the partially inflated sheet in the simple refinement region according to the possibility that the region is over refined after pillowing. Pillowing is conducted on the internal boundary quad set of the partially inflated part, so hexahedra are inserted around the partially inflated part in two directions: along the chords topologically perpendicular to the sheet ( $c_{\perp}$ ) and along the chords topologically parallel to the sheet ( $c_{\parallel}$ ), as illustrated in Fig. 12. Denoting the number of  $c_{\parallel}$  as  $n_{c_{\parallel}}$ , and denoting the number of  $c_{\perp}$  as  $n_{c_{\perp}}$ , the priority of the partially inflated sheet is defined in Eq. (5):

$$p = \frac{1}{n_{c_{\parallel}}} \left( \sum_{i=1}^{n_{c_{\parallel}}} T_{c_i} \right) \sum_{j=1}^{n_{c_{\perp}}} T_{c_j} \quad (5)$$

*Shrink Set Optimization.* For the purpose of mesh quality improvement, the partially inflated part of the sheet, which is the shrink set, is optimized before pillowing. After optimization, the shrink set must still be a part of the sheet, therefore only slot hexahedra and step hexahedra are dealt with in the shrink set optimization procedure. Detailed rules are described in Section 6.4.2. Because two hexahedra are inserted into every chord passing through the shrink set on average, the shrink ratio threshold of simple refinement region is set to 0.3.

## 7. Experimental Results

The proposed algorithm is implemented using C++ based on ACIS geometric engine. Preliminary experimental results are illustrated in Fig. 13, 14 and 15, and the running time and scaled Jacobian after mesh editing are listed in Tab. 1.

Due to the irregular shape of the wedge-shaped model, it is difficult to generate its hexahedral mesh using volume decomposition-based methods [17]. In Fig. 13, a cubic-shaped hexahedral mesh is deformed into a wedge-shaped mesh by direct editing, and its mesh quality is better than the example in [17], whose size-shape metric [18] and scaled Jacobian are illustrated in (c) and (d).

In Fig. 14, the radius of the blind hole is modified. Compared with Sheffer's method, our method is able to successfully obtain the deformed mesh with good mesh quality even if sheets are partially deformed.

In Fig. 15, the shape of sim-courier(simplified courier) mesh model is optimized through direct editing. The detailed model modifications include the modifications of the radius and the length of the centre hole, and material reduction at the connection parts. Geometric faces are rotated when editing the connection parts, so the associated

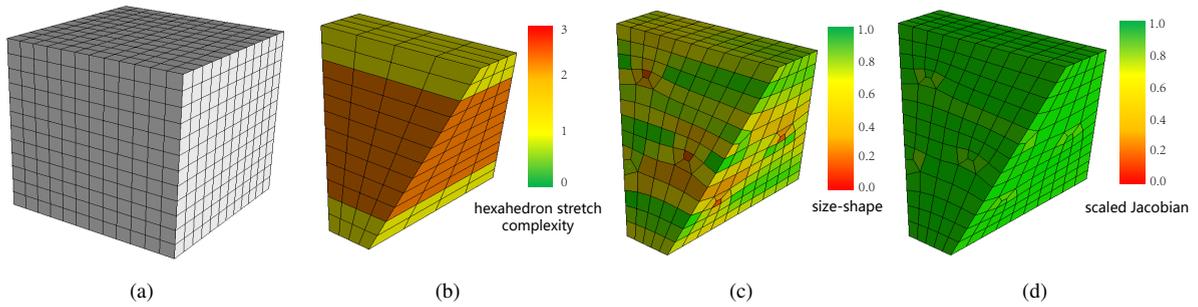


Fig. 13. Direct editing on cubic-shaped mesh: (a)cubic-shaped mesh before direct editing; (b)complex refinement region and simple refinement region; (c)distribution of size-shape metric after direct editing; (d)distribution of scaled Jacobian after direct editing.

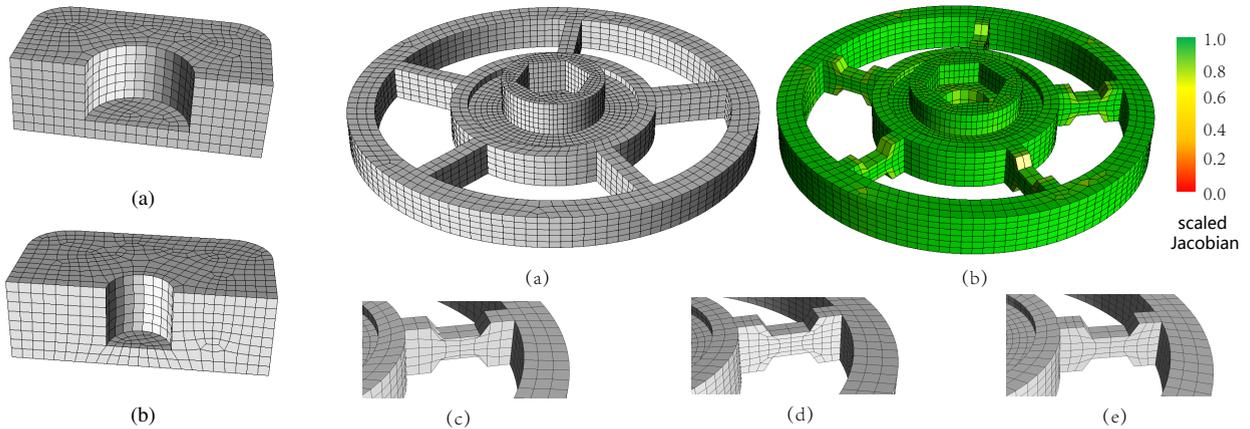


Fig. 14. Direct editing on blind hole: (a)before direct editing; (b)after direct editing.

Fig. 15. Direct editing sim-courier mesh: (a)mesh of sim-courier; (b)distribution of scaled Jacobian after direct editing; (c)connection part after mesh deformation; (d)connection part after fundamental sheets insertion; (e)connection part after coarsening and refinement.

Table 1. Scaled Jacobian after mesh editing and running time

	Hex Num.		Scaled Jacobian				Running Time(s)			
	Before	After	Min.	Max.	Mean	Std.Dev.	Deform	Boundary	Adapt	Total
Cubic	1728	666	0.626	0.994	0.915	0.080	0.520	0.001	10.72	11.24
Sink hole	2354	2360	0.506	1.000	0.949	0.071	0.631	0.000	27.62	28.25
Sim-courier	7393	6114	0.541	1.000	0.935	0.075	2.408	251.2	328.8	582.4

geometric curves are captured by fundamental sheets through sheet insertion and extraction, improving mesh quality at concave curves, as shown in (c),(d) and (e).

### 8. Conclusion and Future Work

In this paper, a novel hexahedral mesh editing method is proposed. By using direct editing technique, mesh editing becomes easier and more flexible. The most remarkable characteristic of our method is being able to get the edited mesh optimized automatically. Specifically, we determine mesh regions that need optimizing by evaluating mesh quality and size with our measuring metrics, and utilize suitable dual operations to effectively conduct optimizations. Based on these dual operations we put forward new local mesh coarsening and refinement methods that can control the mesh size of local regions according to the ideal mesh size. With our method, through direct editing on both CAD model and mesh model, the interoperation of them is also facilitated, and the efficiency of analysis can be improved.

In addition, the method could be extended to work on assemblies or connected volumes as well as other morphed meshes with poorly shaped hexahedra.

There are several problems to be resolved in our future work. In our method, it is supposed that the topology is not changed after direct editing operations while sometimes direct editing may change the model's topology. After mesh deforming using Laplacian smoothing, hexahedra are usually tangled at concave curves. It is difficult to determine reasonable dual operations if a large number of hexahedra are tangled, so we are looking forward to applying other options of smoothing methods. Besides, we are also exploring the feasibility of parallelization through GPUs of the mesh smoothing and the computation of chord stretch metric which is updated after each dual operation.

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