A Metric for Automatic Hole Characterization

German Sanchez T[1], John W. Branch[2] and Pedro Atencio[3]

[1] Professor Faculty of Engineering, Systems Engineering Program, Magdalena University, Colombia, E-mail: gsanchez@unimagdalena.edu.co

[2] Professor school of system and informatics, faculty of minas. National University of Colombia - Medellin, Colombia, E-mail: jwbranch@unal.edu.co

[3] Student of system and informatics school, faculty of minas. National University of Colombia - Medellin, Colombia, E-mail: petriu16@gmail.com

Summary: The correct repair of three-dimensional models is still an open research problem, since acquiring processes (methods and technology) still have limitations. Although a wide range of approaches have been proposed, the main limitation is that user intervention is required to decide which regions of the surface should be corrected. We propose an automatic method for hole characterization enabling the classification of real and false anomalies without user intervention by using an irregularity measure based on two geometrical estimations: the torsion contour's estimation uncertainty, and an approximation of geometrical shape measure surrounding the hole.

Key Words: Curve torsion, entropy contour, Integration, surface reconstruction.

1. Introduction

The shape reconstruction process requires estimating a mathematical representation of an object's geometry using a measured data-set from the object [1]. Since there is no an ideal sensor that does not alter the samples obtained, the process should deal with general problems in order to generate models as accurate as possible.

In context, there are many measuring drawbacks in the acquisition step: topological characteristic of the objects, sensor structure, physical properties of the object's material, illumination conditions, among others. These inadequacies represent the main source of anomaly generation, and must be repaired in order to create a valid digital model [2].

The anomalies could be classified into three types: noise, holes artifacts and redundancy. Typically, these anomalies are repaired in a phase called Integration [3]. Whatever the anomaly type is, the process to correct it corresponds to a wide studying area, with many proposed techniques. However, correcting an anomaly is still considered an open problem inside the Computer Vision and Computer Graphics Community. The difficulty lies, in some cases, in the fact that exact nature of the anomaly's source is undetermined or unknown, i.e. the noise distribu-

tion function [4], or its modeling is complex and does not have an unique solution, i.e. the filling of holes.
The classical reconstruction methods need to apply a post-processing procedure after the final stage of surface-fitting. This is mainly due to the difficulty in differentiating the nature of the discontinuity, that is, whether or not it belongs to the actual surface (see Figure 1).

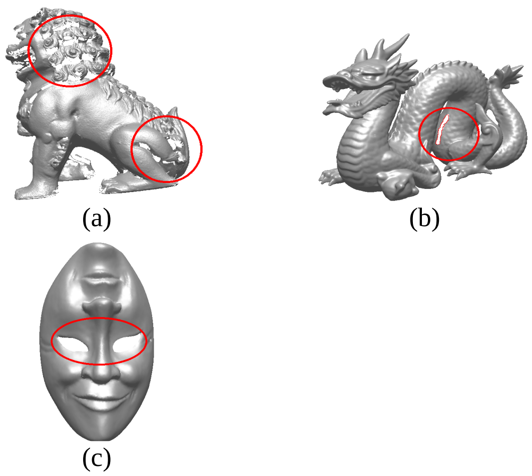

Figure 1. Examples of surface discontinuity, a-b) real discontinuities, c) false discontinuity.

One of the main desirable features in surface reconstruction methods is the ability to fill holes or to complete missing regions. Lack of information is caused mainly by the reflective properties of the material, or by occlusion problems in regions inaccessible to the sensor.
Some methods make an implicit correction during the fitting phase by means of global reconstruction [1] [5]. This approach has two disadvantages: First, it does not permit to keep or represent false holes, *i.e.* those belonging to the object, and second, the quality of the portion generated depends on the technique used and not on the analysis of the intrinsic geometry of the object. While taking the surface as continuum and complete, these techniques reproduce visually appropriate solutions. However, the correction of these anomalies is still limited to particular cases when objects are closed.
A wide range of works has been proposed, many of them can be classified according to the representation used in the initial data, such as mesh repair techniques and dispersed data. However, other classifications can be made according to the conceptual nature of the used technique: based on geometry, algebraic methods and implicit functions.

Hole Detection

For the process of identifying holes, the most simple and functional procedure, with great ease of implementation consists in the idea that a discontinuity an the surface is easily defined as a succession of boundary edges. A boundary edge is defined as a triangle edge that is not shared by any other triangle. The procedure begins with a general search on the mesh to find an initial boundary triangle [6]. The search continues with the neighboring triangles looking for the next boundary edge which must shared a vertex with the initial edge. The process goes on, until the initial triangle is reached. Finally, a closed path that defines the hole contour is traced.

The most important weakness of this simple procedure is that is limited to detecting any discontinuities but does not differentiate between real and false holes, because it assumes a whole closed object surface.

Filling Holes

After the identification procedure is applied, the hole-filling procedure continues by means of surface segment generation, for which, different techniques have been proposed [7] [8] [9] [10] [11] [12] [13] [14]. There are two general trends in the group of geometry-based techniques: repair based on triangles meshes and repair based on voxels. Liepa [6] describes a method for filling holes in meshes based on Delaunay Triangulations of the hole, after a refinement approach to mesh optimization by minimizing an energy functional related to the neighbor curvature estimation. Branch J, [14] filled the holes basing on the local interpolation of the radial basis function. A new segment is generated over a regular and symmetric mesh taking the original point-set density. This approach correctly reproduces the general geometry of the neighborhood hole contour but it fails when a planar surface contains the hole. Wei [15] proposed an algorithm for filling holes that starts with a hole identification phase and then applies a triangulation of the hole region using the Advancing Front Method. Finally, by solving a Poisson equation, the vertex of the generated triangles is adjusted. Although adequate visual results are obtained with this procedure, it is time costly and depends on the size of the hole.

Voxel-based approaches estimate an initial surface volumetric representation by voxel-set. These voxel units are marked with a sign according to their relative position to the surface, that is, inside or outside the surface. Different techniques have emerged to fill the hole in the volumetric space. Curless [16] proposed a method based on volumetric diffusion consisting of a distance function estimation which is used to mark the voxel, and then diffusion is applied through the volume to find the zero-set that define the surface. A similar approach is proposed by Davis [8]. Ju [17] proposed a method of contour surface reconstruction by marking the voxel using an Octree data structure. The procedure is able to fill small gaps, taking into account geometric characteristics. The main limitation is that the hole size must be smaller than the relative surface size. Similarly, Joshua [10] describes an algorithm for hole-filling based on space decomposition by atomic volume regions and defines the model as the union of inner atomic units using a graph cut. User intervention is required to select the way to fill the hole. Chun [18] describes a two-phases approach to 3D model repair. The first phase, a radial basis function interpolation is used to fill the region inside the hole; the second one is a post-processing stage to refine the geometrical detail. In the refinement stage the normals are adjusted to produce adequate visual results.

We can conclude that hole-repairing methods are typically part of a pre-process of the surface fitting step, to get an accurate representation of the geometrical characteristics. In order to correctly fill those holes, a manual-assisted correction procedure is needed. Generally, this manual-assisted procedure has been based on both geometric and mathematical concepts. Their most important weakness lies in its application-domain limited nature, mainly due to their narrow flexibility in objects with different topologies.

In this paper, we propose a metric for robust hole characterization in 3D models. This metric intends to characterize holes through geometric features measures. Our hypothesis is based on a widely accepted definition [19]: free-form objects are smooth except in regions that represent specific geometric details. Hence, if there are not any problems in the acquisition stage, a false contour anomaly should not have large geometric variations; otherwise, it could be caused by problems in the acquisition stage and constitute an anomaly to be repaired.

Thus, if there were any problems during the acquisition process then the data is altered introducing distortion that should not be equal for the segments that define the anomaly. That is, acquisition problems introduce some degree of "contour distortion”. The characterization of each anomaly is based on the quantification of this distortion, which for this particular case is approximated by a quantification of the entropy in the boundary geometry.

The paper is organized as follows: section 2 introduces the topic of anomalies characterization; section 3 describes the hole context irregularity measure; section 4 describes the proposed contour’s irregularity; and section 5 presents the experimental design and results.

2. Anomalies Characterization

The main problem for the automatic repair of holes is to differentiate whether or not the discontinuity belongs to the object. In this context, the key difference between 3D scattered data and other data types such as images or video, is that 3D scattered data are typically irregularly sampled. The points’ distribution of vertices across the surface is not uniform, so to quantify a measure it must be robust under different distribution of points.

In order to get the most accurate estimation of the irregularity of a hole we propose a metric that measures the hole´s most important geometrical features from two points of view: surface irregularity around the hole, and contour curve irregularity measure from the torsion and curvature entropy. A summary of the general diagram is shown in Figure 2.

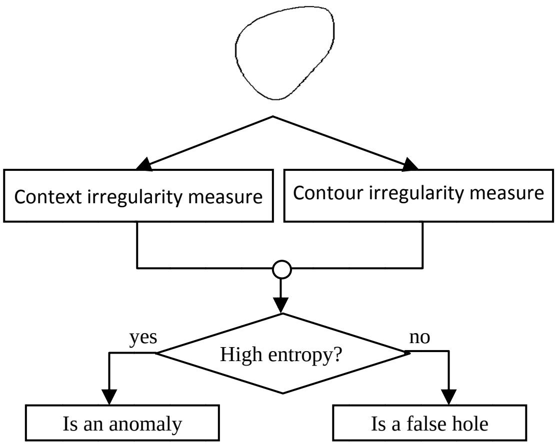

Figure 2. Diagram of hole characterization.

3. Hole's context irregularity measure

Initially, it starts making a cluster environment analysis by regions variability quantification around the hole. For each one of clusters, the centroid is taken to estimate a shape description by geometrical-topological properties. Therefore, this stage implements a general technique of clustering by region growing. The algorithm starts calculating a prior estimation of curvature at each point of surface [6]. It is approximated by principal component analysis obtained resolving the covariance matrix, so that:

$$mc = \frac{1}{n-1}\sum_{i=1}^{n}(p_i - \bar{p})(p_i - \bar{p})^T \qquad (1)$$

where n is the neighborhood size of p, and $\bar{p}$ is the centroid of each cluster and is defined following equation:

$$\bar{p} = \frac{1}{n}\sum_{i=1}^{n} p_i \qquad (2)$$

The curvature estimation is approximate by eq. 3, like in [26]:

$$c = \frac{\lambda_0}{\lambda_0+\lambda_2+\lambda_3}, \text{ where } \lambda_0 \le \lambda_2 \le \lambda_3 \qquad (3)$$

Note that the eigenvector associated with λ_0 indicates the low variation direction of the data, therefore, aims to approximate the normal vector of the surface in the point p, so that c indicates quantitatively the variation of the surface tangent plane, and it constitutes a measure of the variation of the surface.

Once the approximation of the curve is obtained, the next step is to compute a clustering of surfaces, so that the variance of each cluster does not exceed a fixed threshold β. The general description of this procedure is presented in Algorithm 1 and an example in Figure 3.

Algorithm 1: Clustering and center selection of environment.

```
Center selection ( )
Take a Pk random seed point and create a new cluster Ci.
while(Nota ll point are clustering)
Add a vertex Vk ϵ Nci successively to C, while Vci < β, where Nci is a radi-
al neighborhood of Ci and Vci is the estimation of variance of cluster Ci.
Endwhile
```

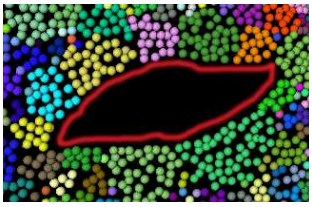

Figure 3. Cluster contour result.

In this work we only use the curvature approximation for clustering the point set around the hole-contour. However, the classical surface curvature measures, such as the Gaussian and the mean curvature at a point of a surface, are not very indicative of local shape. Hence, we used a measured independent of the size of the surface so it is locally described as shape of the surface. The shape index [20] is a measure that describes the second order structure of the surface in the neighborhood of any one of its points. This is thus estimated as follow:

$$s = \frac{2}{\pi}\arctan\frac{k_2 + k_1}{k_2 - k_1} \quad (4)$$

Where $k_1 \geq k_2$ are the *principal curvatures*, and $s \in [-1, +1]$ except for the planar region that has an indeterminate shape index. In order to obtain a global hole-contour description, an average shape-index of cluster is estimated, due to the fact that shape-index is a point measure. So, the centroid shape-index of each cluster *i* is thus:

$$\overline{SI} = \frac{1}{N}\sum_{i=1}^{k} p_i s_i \quad (5)$$

Where, s_i is a shape-index of the cluster c_i, and $c_i \in C$ is the set of k cluster (see Figure 4). The point-set of size $N = p_{1+\cdots+}p_k$, such that p_i is the amount of points of the cluster i. In general, this corresponds to a shape-index average, weighted by the amount of points in each cluster.

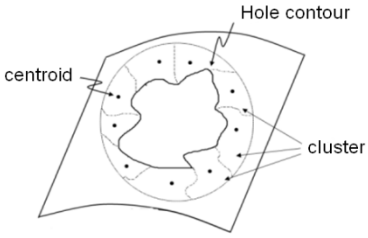

Figure 4. Points' selection for index-shape estimation.

4. Measure of Contour's irregularity

In this step we are interested in measuring the geometrical characteristic of the contour curve. i.e. its means its curvature and its torsion. The aim of these estimations is to quantify its irregularity by means of the uncertainty using the entropy measure.

Contours' torsion Entropy Measure

The two fundamental characteristics of a curve are its curvature and torsion; these allow to measure how a curve bends inside the 3D space, therefore, it constitutes a curve's particular characteristic. Often, we assumed that discontinuous contour curves in smooth objects without acquisition problem are smooth too.

Contour bends give us a measure of irregularity. However, estimating the accurate torsion value of a sample curve, defined by a piecewise linear approximation through an ordered finite collection of points $\{p_i\}$, is not a trivial task since noise is present. i.e., the points p_i stay too close to the curve, but not necessarily lie on it.

In order to approximate a correct classification of contour curves, we used the torsion measure. For spatial curve the torsion is defined by $B'(s) = \tau(s)N(s)$ where $N(s) = r''(s)/\|r''(s)\|$ is the normal vector, s is the arc-length from a specific position $r(t_0)$ given by a parametric curve r, to a close position $r(t_1)$ and defined by $s(t_1) = \int_{t_0}^{t_1}\|r'(u)\|\,du$ (see Figure 5). For a non arc-length rized $r(t)$, $\tau(s)$ is thus estimated:

$$\tau(s) = -\frac{(r' \times r'') \cdot r'''}{\|r' \times r''\|^2} \qquad (6)$$

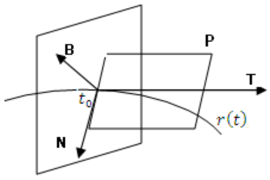

Figure 5. Normal, tangent and osculating plane.

To estimate the torsion we adopt the weighted least squares approach and local arc-length approximation [22] [23] [24]. It considers a samples-set $\{p_i\}$ from a spatial curve. The estimation of derivates of r at p_0 is performed with a point-subset P of $2q + 1$ points such that (see Figure 6):

$$P = \{p_{-q}, p_{-q+1}, \dots, p_q\}.$$

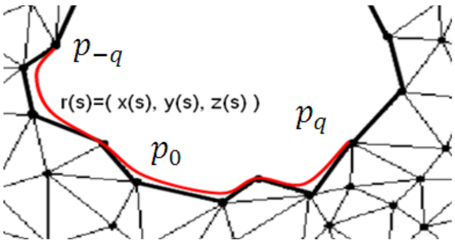

Figure 6. Weight least square curve.

Then a parametric curve $(\hat{x}(s), \hat{y}(s), \hat{z}(s))$ is fitted locally, assuming $p_{0=}r_0$ and an arc-length $s_i$value associated to the samples p_i:

$$\hat{x}(s) = x_0 + x_0' \cdot s_i + \frac{1}{2} x_0'' \cdot s_i^2 + \frac{1}{6} x_0''' \cdot s_i^3$$

$$\hat{y}(s) = y_0 + y_0' \cdot s + \frac{1}{2} y_0'' \cdot s_i^2 + \frac{1}{6} y_0''' \cdot s_i^3 \qquad (7)$$

$$\hat{z}(s) = z_0 + z_0' \cdot s + \frac{1}{2} z_0'' \cdot s_i^2 + \frac{1}{6} z_0''' \cdot s_i^3$$

Taking $\hat{x}$ coordinate, the derivates x_0', x_0'', x_0''' are obtained minimizing [21]:

$$E_x(x_0', x_0'', x_0''') = \sum_{i=-q}^{q} w_i \left(x_i - x_0' s_i - \frac{1}{2} x_0''(s_i)^2 - \frac{1}{6} x_0'''(s_i)^3 \right)^2 \quad (8)$$

Where $w_i = 1$, $s_i = \sum_{k=0}^{i-1} \|p_k - p_{k+1}\|$, $p_i \in \mathbb{R}^3$. It can be written in terms of matrix inversion:

$$A \cdot \mathbb{x} = \mathbb{b} \quad (9)$$

A similar approach is used to estimate the $\mathbb{x}$ and y derivates getting the vectors:

$$\mathbb{Y} = \begin{bmatrix} y_0' \\ y_0'' \\ y_0''' \end{bmatrix} \qquad \mathbb{z} = \begin{bmatrix} z_0' \\ z_0'' \\ z_0''' \end{bmatrix}$$

From the equations system:

$$\begin{cases} A \cdot \mathbb{x} = \mathbb{b} \\ A \cdot \mathbb{y} = \mathbb{b} \\ A \cdot \mathbb{z} = \mathbb{b} \end{cases} \quad (10)$$

here,

$$\begin{bmatrix} a_1 & a_2 & a_4 \\ a_2 & a_3 & a_5 \\ a_4 & a_5 & a_6 \end{bmatrix} \cdot \begin{bmatrix} x_0' & y_0' & y_0' \\ x_0'' & y_0'' & y_0'' \\ x_0''' & y_0''' & y_0''' \end{bmatrix} = \begin{bmatrix} b_{x,1} & b_{y,1} & b_{z,1} \\ b_{x,2} & b_{y,2} & b_{z,2} \\ b_{x,3} & b_{y,3} & b_{z,3} \end{bmatrix}$$

The a_i values and $b_{x,i}$ are defined thus:

$$a_1 = \sum_{i=-q}^{q} w_i s_i^2 \qquad a_2 = \frac{1}{2} \sum_{i=-q}^{q} w_i s_i^3 \qquad a_3 = \frac{1}{4} \sum_{i=-q}^{q} w_i s_i^4$$

$$a_4 = \frac{1}{6} \sum_{i=-q}^{q} w_i s_i^4 \qquad a_5 = \frac{1}{12} \sum_{i=-q}^{q} w_i s_i^5 \qquad a_6 = \frac{1}{36} \sum_{i=-q}^{q} w_i s_i^6$$

$$b_{x,1} = \sum_{i=-q}^{q} w_i s_i \, x_i \qquad b_{x,2} = \frac{1}{2} \sum_{i=-q}^{q} w_i s_i^2 \, x_i \qquad b_{x,3} = \frac{1}{6} \sum_{i=-q}^{q} w_i s_i^3 \, x_i$$

$$b_{y,1} = \sum_{i=-q}^{q} w_i s_i \, y_i \qquad b_{y,2} = \frac{1}{2} \sum_{i=-q}^{q} w_i s_i^2 \, y_i \qquad b_{y,3} = \frac{1}{6} \sum_{i=-q}^{q} w_i s_i^3 \, y_i$$

$$b_{z,1} = \sum_{i=-q}^{q} w_i s_i \, z_i \qquad b_{z,2} = \frac{1}{2} \sum_{i=-q}^{q} w_i s_i^2 \, z_i \qquad b_{z,3} = \frac{1}{6} \sum_{i=-q}^{q} w_i s_i^3 \, z_i$$

Finally, it defines:

$$r_0' = \begin{bmatrix} x_0' \\ y_0' \\ z_0' \end{bmatrix} \qquad r_0'' = \begin{bmatrix} x_0'' \\ y_0'' \\ z_0'' \end{bmatrix} \qquad r_0''' = \begin{bmatrix} x_0''' \\ y_0''' \\ z_0''' \end{bmatrix}$$

The computation of $\tau(s)$ is straightforward, thus:

$$\tau(p_0) = -\frac{(r_0' \times r_0'') \cdot r_0'''}{\|r_0' \times r_0''\|^2}$$

Due to their nature, hole-characterization problems suggest solutions based on inference, since it needs a process of drawing conclusions from available information that is partial, insufficient and that does not allow to reach an unequivocal, optimal and unique solution. Then we need to make inferences from the available data assuming it is noisy. Specifically, the topic of hole characterization constitutes a highly ambiguous example to take decisions because there are many possible configurations of irregular contours. Both aspects, noise and ambiguity imply taking uncertainty in account.

The adequate way to deal with the presence of uncertainty, related to lack of information, is to introduce assumptions about the problem's domain or *a priori* knowledge about the data, by means of the notion of degrees of belief. It should be treated using the classical rules of calculus of probabilities. The rules of probability theory allow us to assign probabilities to some "complex" propositions on the basis of the probabilities that have been previously assigned to other, perhaps more "elementary" propositions. However, in order to estimate a measure to characterize contours, we are not interested in just probability estimation about a geometrical characteristic, but also in its variability. High variability could be measured through entropy. Specifically, conditional entropy is used.

Given two variables x and y, the $S_{x|y}$ quantity that measures the amount of uncertainty about one variable x when we have some limited information about another variable y is conditional entropy [25]. It is obtained by calculating the entropy of x as if the precise value of y were known and then taking the expectation over the possible values of y.

$$S_{x|y} = -\sum_{y} p_y\, S[p_{x|y}] = -\sum_{y} p_y \sum_{x} p_{x|y} \log(p_{x|y})$$

In a similar way,

$$S_{x|y} = -\sum_{x,y} p_{xy} \log(p_{x|y}) \qquad (11)$$

Given a sequence of n points $P\colon p_i \in \mathbb{R}^3$ forming the contour of a 3D curve defining an anomaly, and a set φ of geometrical characteristic measured associated to each one in P. We want to measure the irregularity in φ from a prior knowledge of some geometrical characteristic measure. It means, the certainty a point p_i is estimated taken a l -set $\psi_i\colon\{\varphi_k\colon i-l-1 < k < i-1\}$ over a sorted sequence of points used to estimate the next value. The certainty or in inverse form, the unpredictably of all ψ_i is related to entropy.

$$S_\varphi = -\sum_{i}^{n} p(\varphi_i, \psi_i) \log \left(p(\varphi_i | \psi_i)\right) \qquad (12)$$

Where $\psi_i : \{\varphi_{i-(l+1)}, \varphi_{i-l}, \dots, \varphi_{i-2}, \varphi_{i-1}\}$, and:

$$\varphi_i = \tau(p_i) = -\frac{(r' \times r'') \cdot r'''}{\|r' \times r''\|^2}.$$

Contours' curvature Measure

The principal limitation of a descriptor based on torsion measure is deal with planar curves. Planar curves may appear as a result of occlusions; although these are uncommon the torsion based decision is inappropriate. However, in order to completeness, our metric take into account those cases and use for planar cases a tangent vector variability measure like irregularity.

For anomalies in planar cases the tangent variability usually is high, otherwise, real holes show smooth changes between tangents angles (see Figure 7).

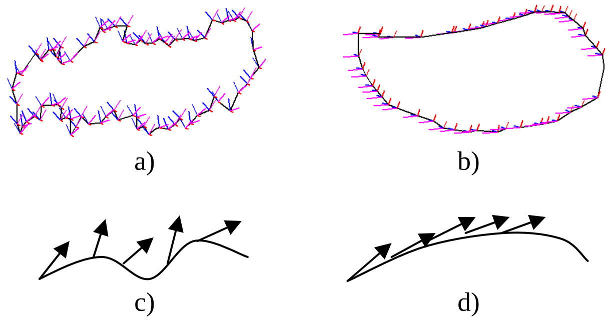

Figure 7. Tangent vector (Red) variability for a) Dragon object and b) eye contour from Mask, c-d) close view of both cases respectively.

To estimate this measure, we take the weighted least squares approach and local arc-length approximation made in the section 4. Tangent vector is defines $T(t) = N(s) \times B(s)$, or in derivates terms $T(t) = \frac{r'(t)}{\|r'(t)\|}$. We estimate the entropy S_T of angle between successive tangents like the equation 12, replacing the torsions distributions by angle between tangents distribution. And finally quantify the global entropy of the contour S_c by:

$$\mathrm{S_c} = \mathrm{S_\varphi} + \mathrm{S_T} \qquad (13)$$

Finally,

$$Irregularity = \|\overline{SI}\|(S_\varphi + S_T)$$

For undefined cases of $\overline{\mathrm{SI}}$ it means for planar case, the $\mathrm{S_t}$ measure is an accurate estimation of irregularity.

5. Experiment Design and Result

In order to estimate the p_{xy} and $p_{x|y}$ quantity given a continuous variable of torsion measure, we used a bin size r to discrete the domain of an experimental set of hole-contour configurations. The experimental set was obtained from 10 images with real hole anomaly and 10 contours of false hole anomaly, in partial and complete 3d models' range data. Some images were scales to maintain a fixed scale. It was done by subtracting the mean and dividing by the standard deviation. The parameter r was set to 10% of standard deviation and m set to 2. The irregularity was estimated with equation 13. Figure 8 shows the irregularity estimated for both sets. It shows that the irregularity measure is highly sensitive to the irregularities in the contour. Determining if an anomaly is real or false is straightforward because the values are sufficiently separated. The midpoint of the range of separation is 3.1. The experimental threshold for classification was estimated in this value.

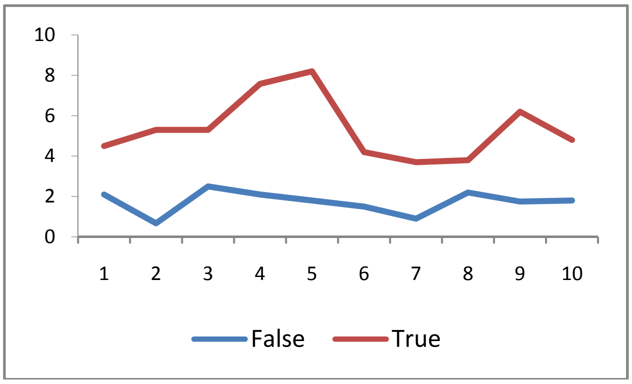

Figure 8. Irregularity values of false (blues) and real anomalies (red).

The irregularity could increase when increased the separation of the data. The method is highly sensitive to noise; small changes in the regularity of the contour show an equal increase in the estimation of entropy. This method can be used as an initial step in the process of correcting anomalies. We aim to complement the method with the filling process to propose an automatic robust method to correct anomalies.

The proposed method allows to estimate a metric for the automatic classification of anomalies in range images. The purpose of the method is to automate a process that has traditionally required user intervention. The method estimates the most relevant geometric characteristics of curves and surfaces to describe them.
The anomalies used as the working set were mostly generated by occlusion.

6. References

[1] Curless, B., New Methods for Surface Reconstruction from Range Images. Stanford University, 1997.

[2] Kumar, Amitesh, et al., "A Hole-filling Algorithm Using Non-uniform Rational B-splines." s.l. : Springer, 2007. pp. 169-182.

[3] Turk, Greg and Levoy, Marc., "Zippered polygon meshes from range images." s.l. : ACM, 1994. pp. 311-318.

[4] Sun, Xianfang, et al., "Noise in 3D laser range scanner data." Shape Modeling and Applications, 2008. SMI 2008. IEEE International Conference on, 2008, pp. 37-45.

[5] Carr, J. C., et al., "Smooth surface reconstruction from noisy range data." s.l. : ACM Press, 2003. pp. 119--ff.

[6] Liepa, Peter., "Filling holes in meshes." s.l. : Eurographics Association, 2003. pp. 200-205.

[7] Dorai, Chitra, et al., "Registration and Integration of Multiple Object Views for 3D Model Construction." IEEE Transactions on Pattern Analysis and Machine Intelligence, 1998, Vol. 20, pp. 83-89.

[8] Davis, James, et al., "Filling Holes in Complex Surfaces Using Volumetric Diffusion." 2002. pp. 428-438.

[9] Wang, J. and Oliveira, M.M., "A hole-filling strategy for reconstruction of smooth surfaces in range images." Computer Graphics and Image Processing, 2003. SIBGRAPI 2003. XVI Brazilian Symposium on, 2003, pp. 11-18.

[10] Podolak, Joshua and Rusinkiewicz, Szymon., "Atomic volumes for mesh completion." s.l. : Eurographics Association, 2005. p. 33.

[11] Bischoff, Stephan, Pavic, Darko and Kobbelt, Leif., "Automatic restoration of polygon models." ACM Trans. Graph., s.l. : ACM, 2005, Vol. 24, pp. 1332-1352.

[12] Guo, Tongqiang, et al., "Filling Holes in Meshes and Recovering Sharp Edges." Systems, Man and Cybernetics, 2006. SMC '06. IEEE International Conference on, 2006, Vol. 6, pp. 5036-5040.

[13] Bendels, G. H., Schnabel, R. and Klein, R., "Fragment-based Surface Inpainting." [ed.] M. Desbrun and H. Pottmann. s.l. : The Eurographics Association, 2005.

[14] Branch, John, Prieto, Flavio and Boulanger, Pierre., "Automatic Hole-Filling of Triangular Meshes Using Local Radial Basis Function." s.l. : IEEE Computer Society, 2006. pp. 727-734.

[15] Zhao, Wei, Gao, Shuming and Lin, Hongwei., "A robust hole-filling algorithm for triangular mesh." Vis. Comput., s.l. : Springer-Verlag New York, Inc., 2007, Vol. 23, pp. 987-997.

[16] Curless, Brian and Levoy, Marc., "A volumetric method for building complex models from range images." s.l. : ACM Press, 1996. pp. 303-312.
[17] Ju, Tao., "Robust repair of polygonal models." s.l. : ACM, 2004. pp. 888-895.
[18] Chen, Chun-Yen and Cheng, Kuo-Young., "A Sharpness-Dependent Filter for Recovering Sharp Features in Repaired 3D Mesh Models." Visualization and Computer Graphics, IEEE Transactions on, 2008, Vol. 14, pp. 200-212.
[19] Besl, P.J., The free-form surface matching problem, in Machine Vision for Three-Dimensional Scenes. s.l. : Academic Press, 1990.
[20] Koenderink, J. J. and van Doorn, A. J., "Surface shape and curvature scales. ." Butterworth-Heinemann Newton, MA, USA : Image Vision Comput. pp. 557-565. , 1992, Issue 8, Vol. 10. ISSN:0262-8856.
[21] M., do Carmo., Differential geometry of curves and surfaces. Englewood Cliffs : NJ: Prentice Hall, 1976.
[22] Thomas Lewiner, Joao D. Gomes Jr., Helio Lopes, Marcos Craizer., "Curvature and torsion estimators based on parametric curve fitting." s.l. : Computers & Graphics, pages 641-655, 2005, Issue 5, Vol. 29. ISSN 0097-8493.
[23] P. Lancaster and K, Salkauskas., "Surfaces generated by moving least squares methods." s.l. : Math. Comp, p141-158, 1981, Issue 155, Vol. 37.
[24] P. Lancaster, k. Salkauskas., "Curve and Surface Fitting: An Introduction." s.l. : Academic Press, 2002.
[25] Ariel, Caticha., "Lectures on Probability, Entropy, and Statistical Physics." Albany : Department of Physics, University at Albany , 2008. 0808.0012.
[26] Pauly, Mark, Gross, Markus and Kobbelt, Leif P., "Efficient simplification of point-sampled surfaces.", IEEE Computer Society, 2002. pp. 163-170.